6.094 Introduction to MATLAB®
January (IAP) 2009

Homework #4: Advanced Methods

Exercise 1. Generating Distributions

Do Problem 14 and 15 in Chapter 7 on page 460 of Palm.

Exercise 2. Simulating Stock Market

1. Do Problem 19 in Chapter 7 on page 460-461 of Palm. Include a histogram of all runs.
2. Recent stock market events have you concerned about the effects of black swans, highly unlikely yet high impact events resulting from skewed probability distributions. Simulate the effect of a black swan.

   Namely, assume that there is a .1 percent chance (1 in 1000) every day of a market crash. This market crash would permanently lower the stock's mean by 3 standard deviations (15). This can be simulated by generating a random number for each day with `rand()`. In the event that the number generated for a day is ever smaller than .001, subtract 15 from every subsequent day's stock price. You can choose however you'd like to implement this. Now what is the mean, standard deviation, minimum, and maximum profit of the strategy? Run at least 1000 trials, and include a histogram.

Exercise 3. Monte Carlo Estimation of π

We'll make a simple Monte Carlo simulation to estimate π. Imagine a circle of radius 1 inscribed inside a square.

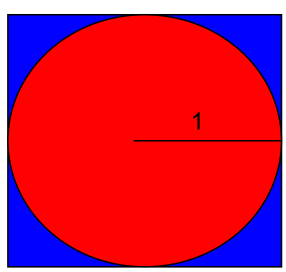

Figure 1: Circle Inscribed by Square

If we randomly shoot arrows at this square target, we expect that the ratio for the number of arrows landing in the circle to the number of arrows landing in the entire square to be,

$$\frac{\sharp \text{ arrows in circle}}{\sharp \text{ arrows in square}} = \frac{\pi^2 1^2}{2^2} = \frac{\pi}{4} \tag{1}$$

.

Therefore, we can estimate the value of π by simulating where our random arrows land.
To do this, pick a random vector (size $n \times 1$) of x values, and a random vector (size $n \times 1$) of y values on the range $[-1, 1]$. Then calculate the distance of each x, y pair from the origin. If this distance is less than or equal to 1, the arrow falls in the circle. Note that when the problem is structured this way, ALL the arrows will fall within the square, so the estimate of π is,

$$\pi \approx 4 \frac{\sharp \text{ arrows in circle}}{\sharp \text{ arrows in square}} \tag{2}$$

For each part below, set the state of the random number generator to 0 before doing the simulation.

1. Carry out the above calculation for various sample sizes (n from 1 to $1,000,000$), and plot the absolute error with a logarithmic y axis. You can do this without having to generate a new vector of random numbers for each n and without using loops.

2. On the same axes as in the first part, plot the result of the same calculation, except that now, run the simulation on a circle of radius 1 within a box with side length 4, as illustrated below (note that now, $\pi \approx 16 \frac{\sharp \text{ arrows in circle}}{\sharp \text{ arrows in square}}$). Which experiment converges faster?

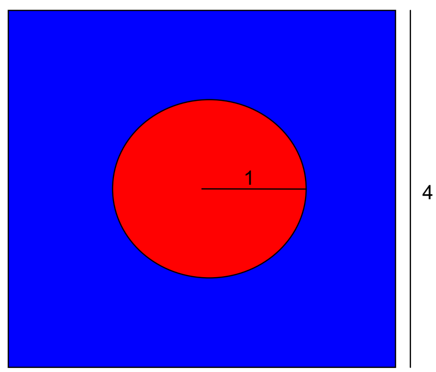

Figure 2: Circle Inscribed by Larger Square

In general, Monte Carlo simulations can be used to solve hard deterministic problems, but the setup of the simulation, the probability distribution from which you draw numbers, and the quality of the pseudorandom number generator can all affect the accuracy of the result.

Exercise 4. Symbolic Manipulation

Do Problem 4 in Chapter 10 on page 637 of Palm.

Exercise 5. Symbolic Equation Solving

Do Problem 8 in Chapter 10 on page 637 of Palm. You will find the symbolic function `solve`.

Exercise 6. Symbolic Linear Algebra

Do Problem 49 in Chapter 10 on page 647 of Palm.