Technical University
of Iasi, Romania

Faculty of Electronics and
Telecommunications

Signals, Circuits and
Systems laboratory

Circuite neliniare pentru prelucrarea semnalelor

Prof. Victor Grigoras

Cuprins

- Ecuatii de stare neliniare
- Dinamica neliniara
  - Dinamica de tip constant
  - Dinamica periodica
  - Dinamica de tip cuasiperiodic
  - Dinamica haotica
- Aplicatii ale dinamicii neliniare
  - Aplicatii in optimizare
  - Aplicatii in sisteme adaptive
  - Aplicatii in modulatie
  - Aplicatii in criptare

Bibliografie

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- *I. Ciocoiu, V. Grigoras; "Tehnici moderne de procesare a semnalelor"*

Ed. Cermi 2005 (cap. 4 si 5):

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- *T. S. Parker, L. O. Chua; "Chaotic Dynamics Algorithms" 1989:*

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- *H. G. Schuster; "Deterministic Chaos" 1984*
- *J. Guckenheimer, P. Holmes; "Nonlinear Oscillations, Dynamical Systems and Bifurcations of Vector Fields" Springer-Verlag 1983*

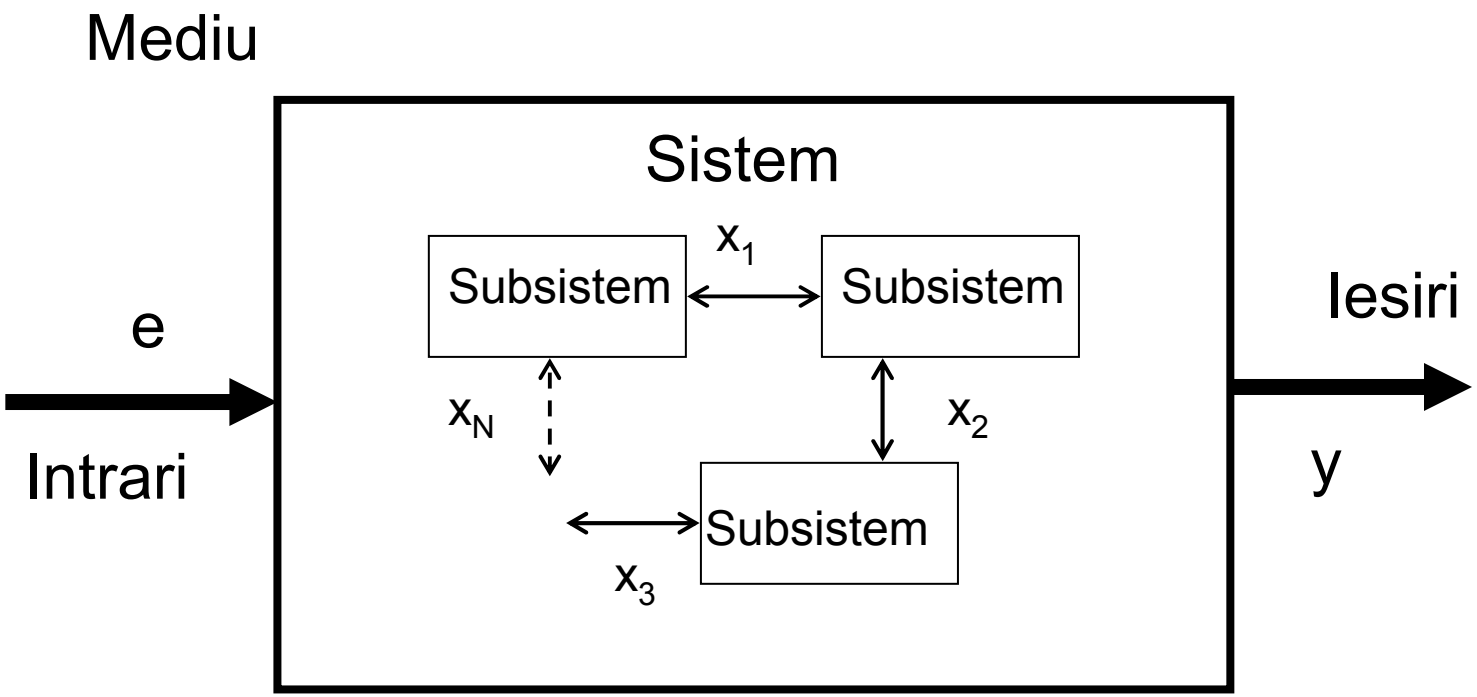
Ecuatii de stare neliniare

- Abordarea **sistemica**
- Prezentare in paralel pentru cazurile: **analogic si discret**
- Caracterizare **intrare – iesire**
- Caracterizare intrare – **stare** - iesire
- Notiunea de **stare**
- **Avantaje** in caracterizarea circuitelor neliniare

Abordarea sistemica

- Delimitarea sistemului
  - Partitionarea universului de discurs in **sistem** si **mediu**
- Precizarea **intrarilor**
  - Influenta mediului asupra sistemului
- Precizarea **iesirilor**
  - Influenta sistemului asupra mediului
- Evidentierea **structurii** interne:
  - **Elemente** de sistem/circuit
  - **Interconexiunile** intre elemente
  - Precizarea variabilelor de **stare**

Abordarea sistemica (cont.)



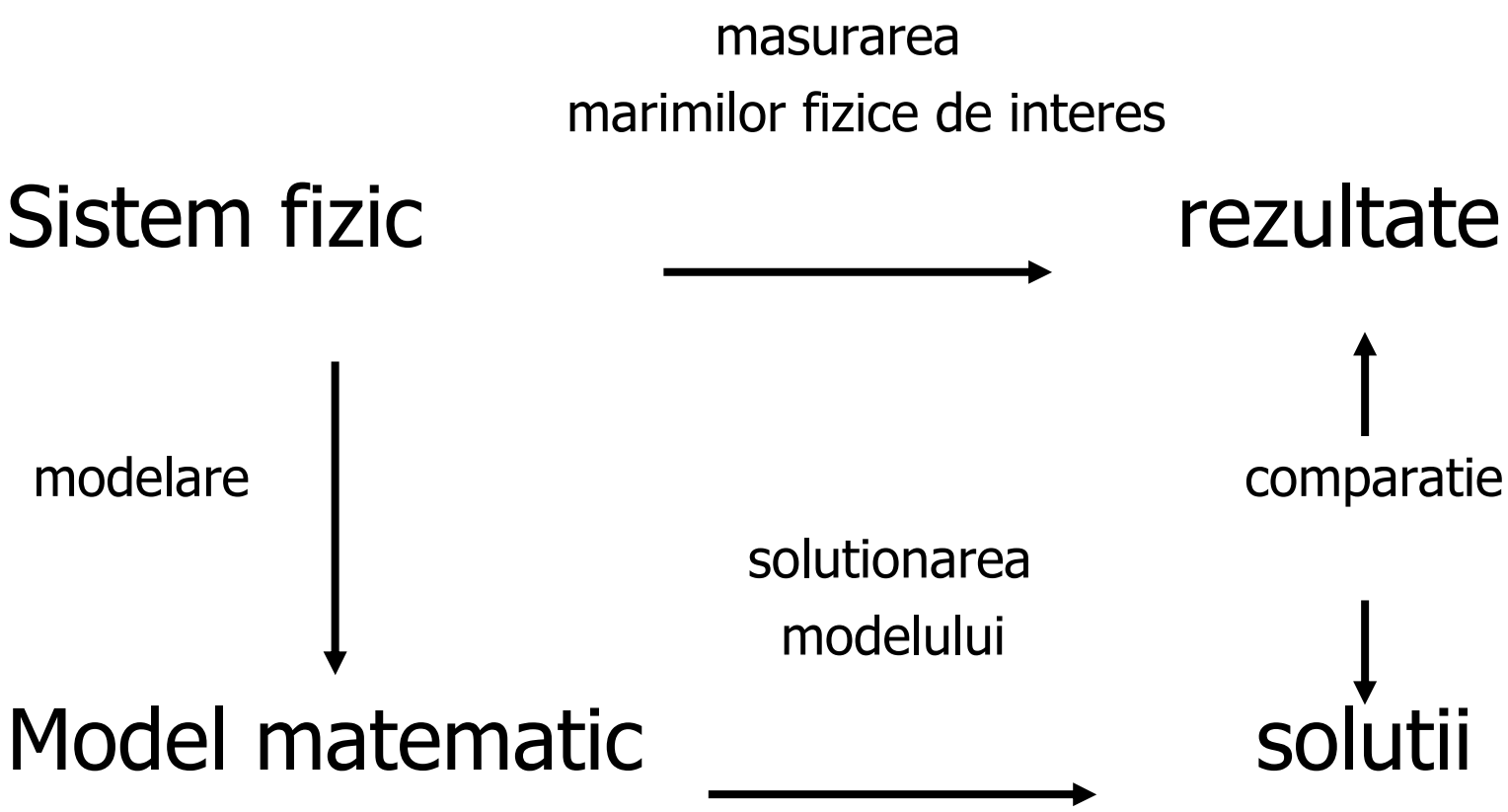
Abordarea sistemica (cont.)

Sistem fizic

modelare ↓

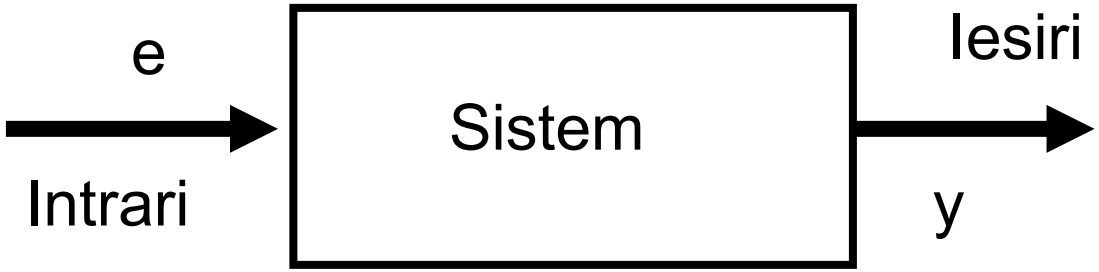
Model matematic $\xrightarrow{\text{solutionarea modelului}}$ solutii

Abordarea sistemica (cont.)



Caracterizarea intrare-iesire

- Dupa modelarea sistemului fizic, nu se mai studiaza structura acestuia



- implicit, nu se mai studiaza dinamica interna a acestuia

Ecuatii analogice

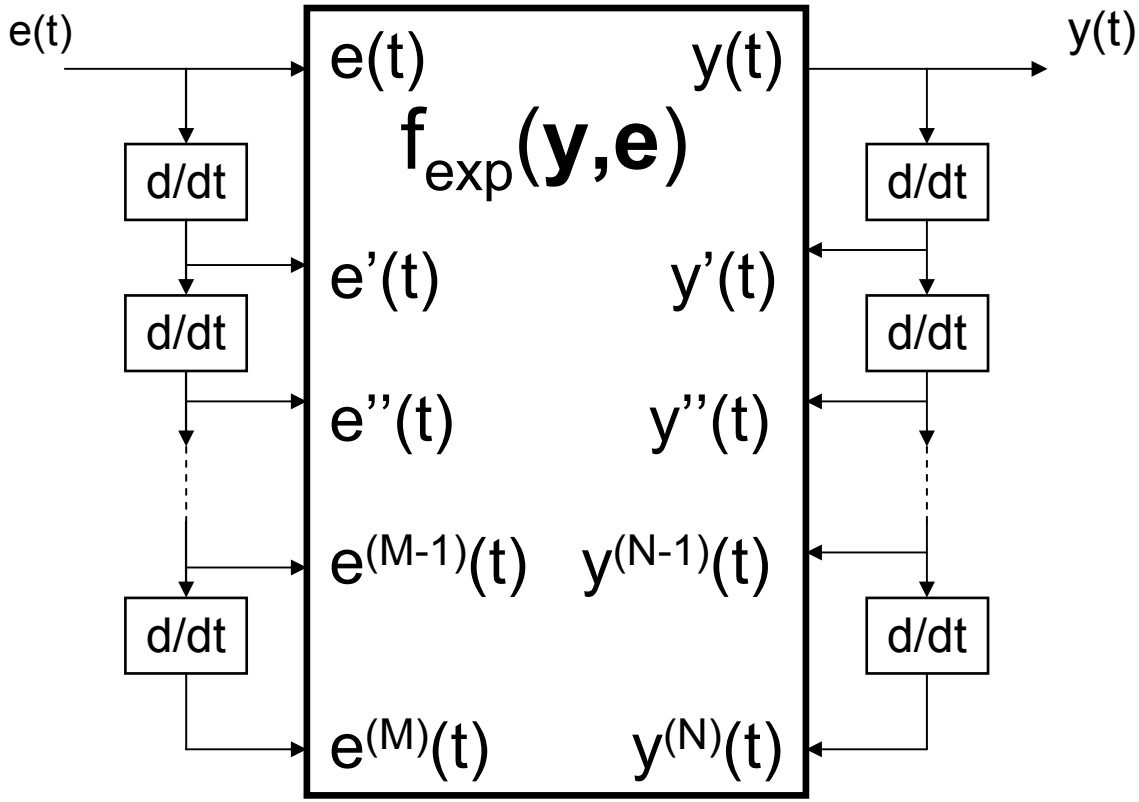
- Sistemele analogice pot fi caracterizate de o singura ecuatie diferentiala de ordinul N continand numai semnalele de intrare si iesire:
  - In forma implicita (cea mai generala)

- S: $$f_{imp}\left(y(t), y'(t), \ldots, y^{N}(t), e(t), e'(t), \ldots, e^{M}(t)\right) = 0$$

  - Ordinul sistemului: N >= M
  - In forma explicita (numai in cazurile in care se poate face explicitarea)

- S: $$y(t) = f_{\exp}\left(y'(t), \ldots, y^{N}(t), e(t), e'(t), \ldots, e^{M}(t)\right)$$

Schema bloc de implementare a ecuatiei explicite



Ecuatii discrete

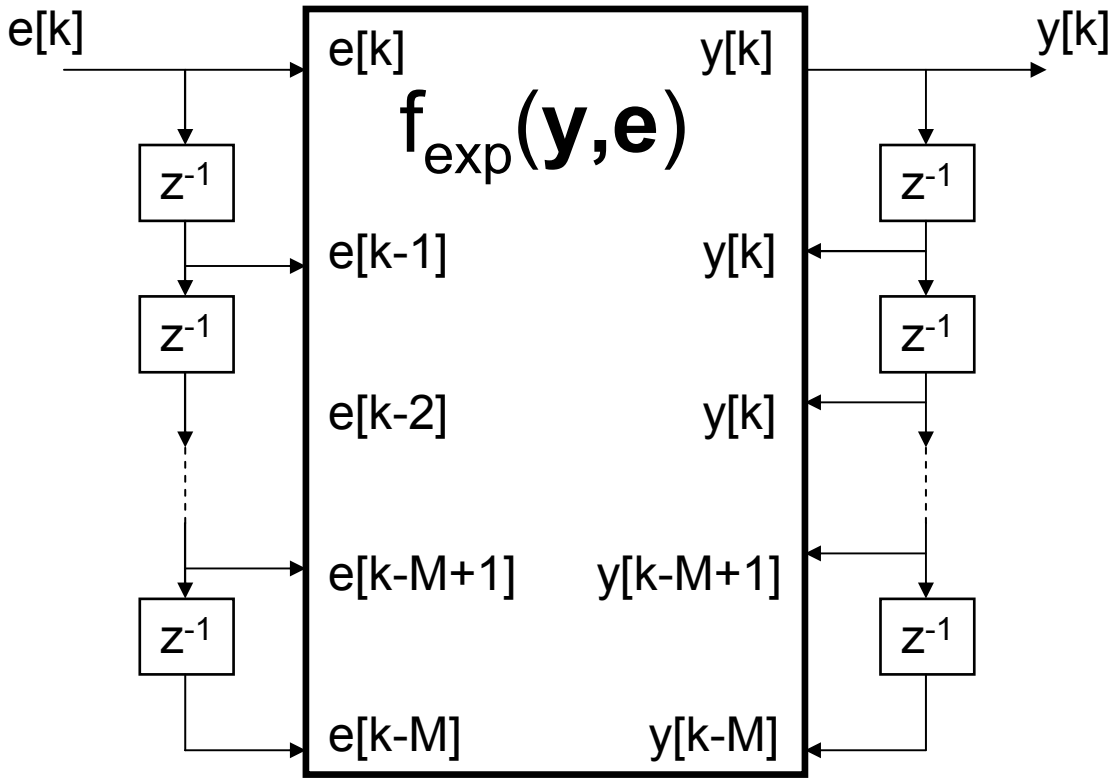
- Sistemele discrete pot fi caracterizate de o singura ecuatie cu diferente, de ordinul N, continand numai semnalele de intrare si iesire:
  - In forma implicita (cea mai generala)

$$f_{imp}\left(y[k], y[k-1], \ldots, y[k-N], e[k], e[k-1], \ldots, e[k-M]\right) = 0$$

  - Ordinul sistemului: N < > M
  - In forma explicita (numai in cazurile in care se poate face explicitarea)

$$y[k] = f_{\exp}\left(y[k-1], \ldots, y[k-N], e[k], e[k-1], \ldots, e[k-M]\right)$$

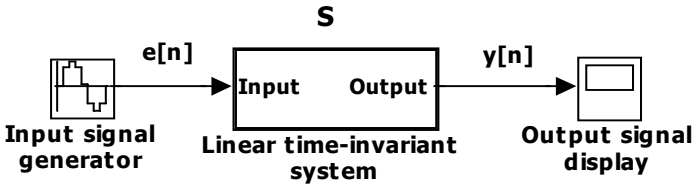
Schema bloc de implementare a ecuatiei explicite



Liniaritatea

- Modelam matematic un sistem, ca un operator pe un spatiu vectorial de semnale:

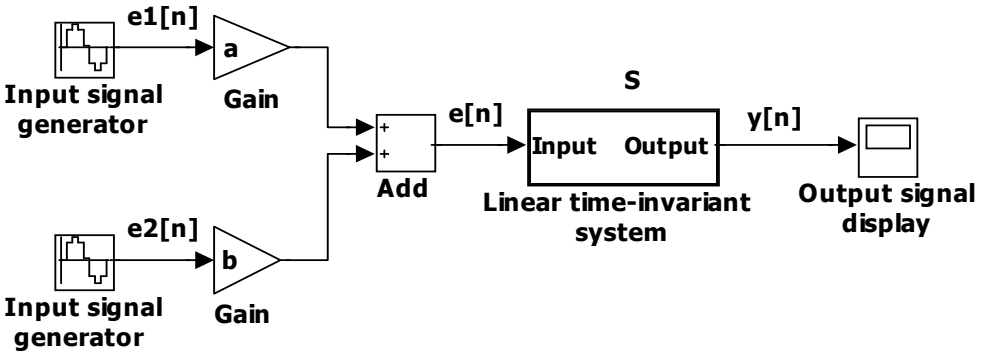
$$S: V \rightarrow V \qquad y = S(e)$$

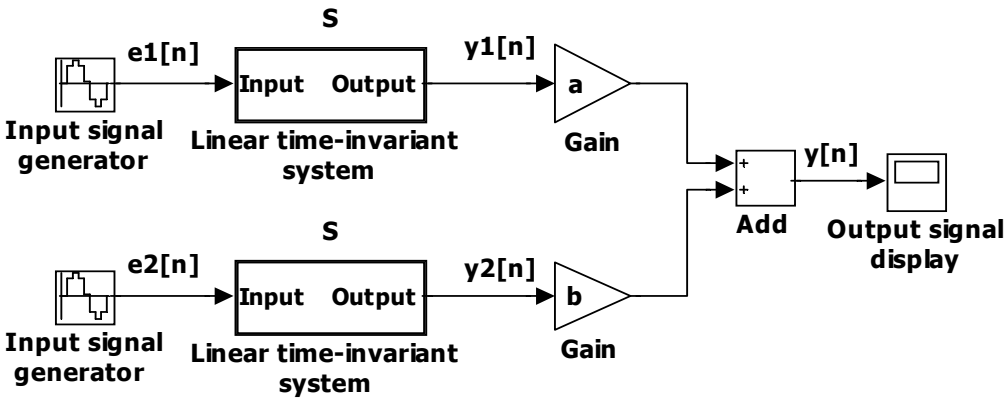


- Principiul suprapunerii efectelor:

$$S(a_1 \cdot e_1[n] + a_2 \cdot e_2[n]) = a_1 \cdot Se_1[n] + a_2 \cdot Se_2[n]$$

Liniaritatea (cont.)





Liniaritatea (cont.)

- Liniaritatea este compusa din doua proprietati elementare:
- Aditivitatea:

$$e = e_1 + e_2 \quad \Rightarrow \quad y = y_1 + y_2$$

$$y = S(e); \quad y_1 = S(e_1); \quad y_2 = S(e_2)$$

- Omogenitatea: $e = a \cdot e_1 \quad \Rightarrow \quad y = a \cdot y_1$

$$y = S(e); \quad y_1 = S(e_1)$$

- Global: $e = a_1 \cdot e_1 + a_2 \cdot e_2 \quad \Rightarrow \quad y = a_1 \cdot y_1 + a_2 \cdot y_2$

$$y = S(e); \quad y_1 = S(e_1); \quad y_2 = S(e_2)$$

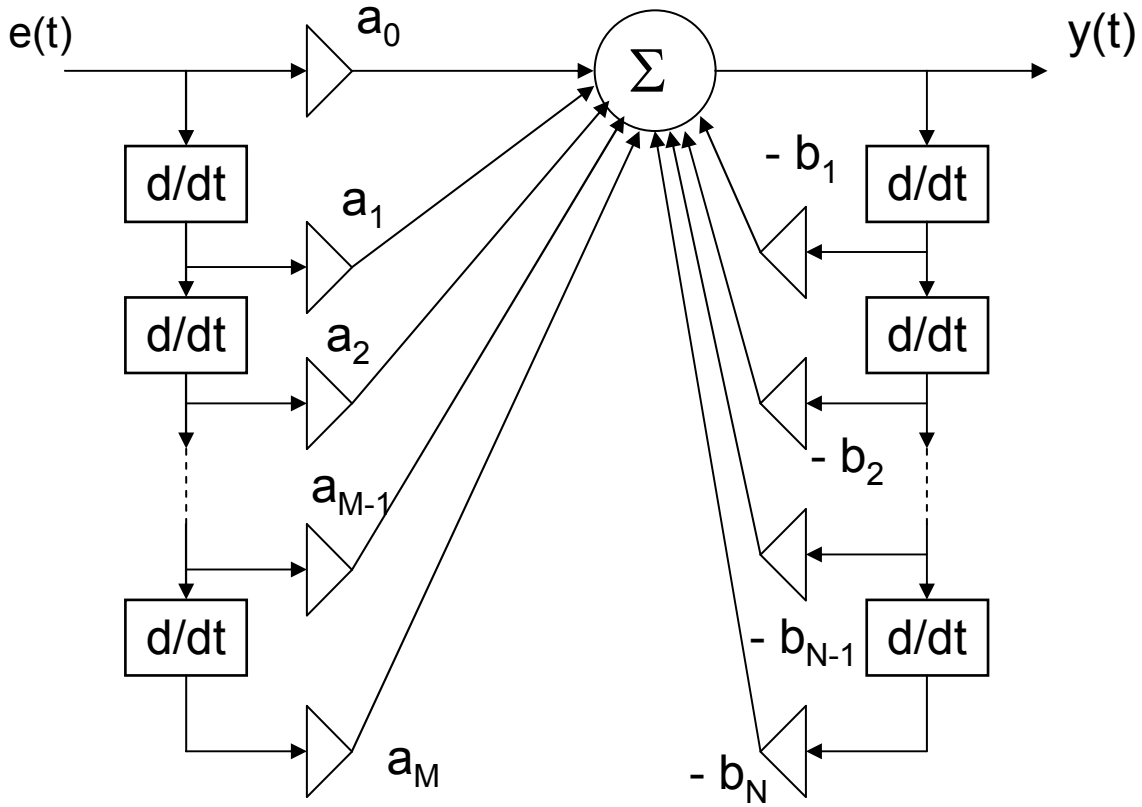
Sisteme liniare analogice

- In ecuatiile analogice se inlocuiesc functiile algebrice neliniare cu combinatii liniare cu coeficienti constanti

$$\left.\begin{aligned} &\sum_{n=0}^{N} b_n \cdot y^{(n)}(t) - \sum_{m=0}^{M} a_m \cdot e^{(m)}(t) = 0 \\ &b_0 = 1 \end{aligned}\right\} \Rightarrow$$

$$y(t) = \sum_{m=0}^{M} a_m \cdot e^{(m)}(t) - \sum_{n=1}^{N} b_n \cdot y^{(n)}(t)$$

Schema bloc de implementare analogica



Caracterizrea frecventiala analogica

- Sistemul fiind **liniar si invariant** in timp, se poate aplica transformata **Laplace** ecuatiei diferentiale:

$$\sum_{m=0}^{M} a_m \cdot s^m \cdot E(s) = \sum_{n=0}^{N} b_n \cdot s^n \cdot Y(s)$$

- Se observa ca transformata Laplace transforma ecuatia diferentiala intr-o **ecuatie algebrica**;
- Din ecuatia algebrica se poate deduce **functia de transfer**:

$$H(s) = \frac{Y(s)}{E(s)} = \sum_{m=0}^{M} a_m \cdot s^m \Big/ \sum_{n=0}^{N} b_n \cdot s^n$$

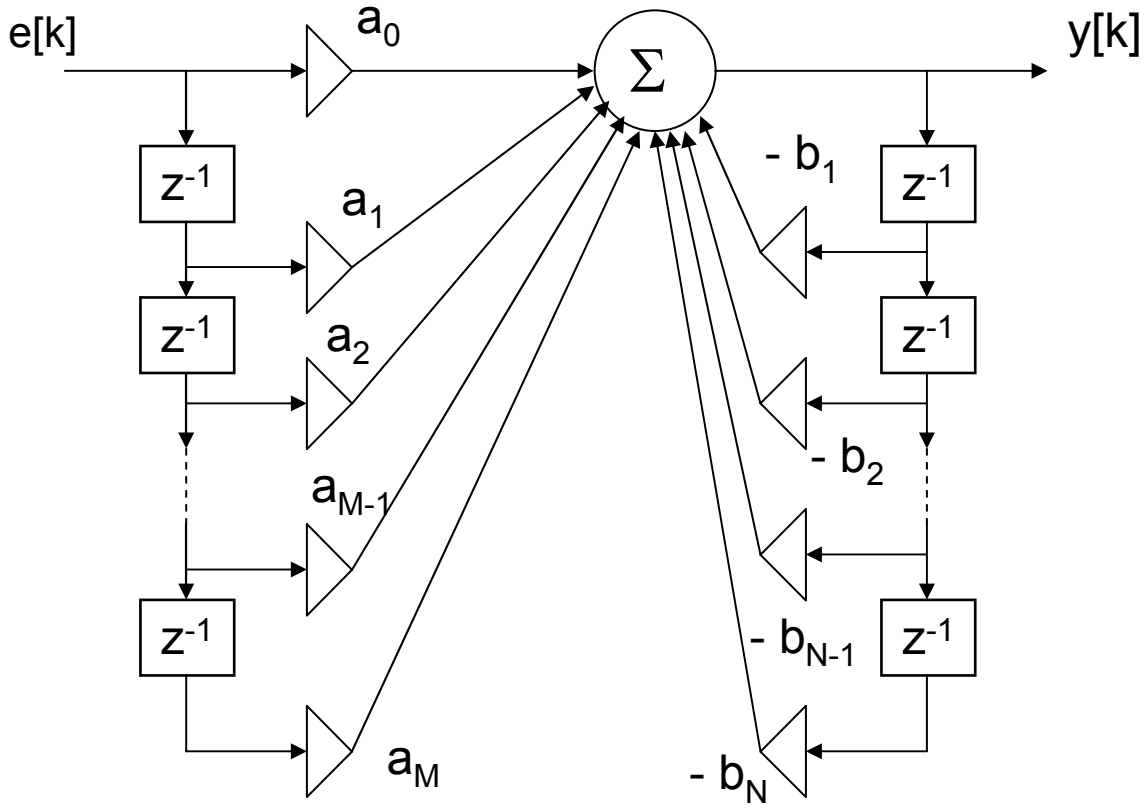
Sisteme liniare discrete

- In **ecuatiile cu diferente** se inlocuiesc functiile algebrice neliniare cu doua combinatii liniare cu coeficienti constanti:

$$\left.\begin{aligned} &\sum_{n=0}^{N} b_n \cdot y[k-n] - \sum_{m=0}^{M} a_m \cdot e[k-m] = 0 \\ &\qquad\qquad\qquad\qquad\qquad\qquad b_0 = 1 \end{aligned}\right\} \Rightarrow$$

$$y[k] = \sum_{m=0}^{M} a_m \cdot e[k-m] - \sum_{n=1}^{N} b_n \cdot y[k-n]$$

Schema bloc de implementare discreta



Caracterizrea frecventiala discreta

- Sistemul fiind **liniar si invariant** in timp, se poate aplica transformata **Z** ecuatiei cu diferente:

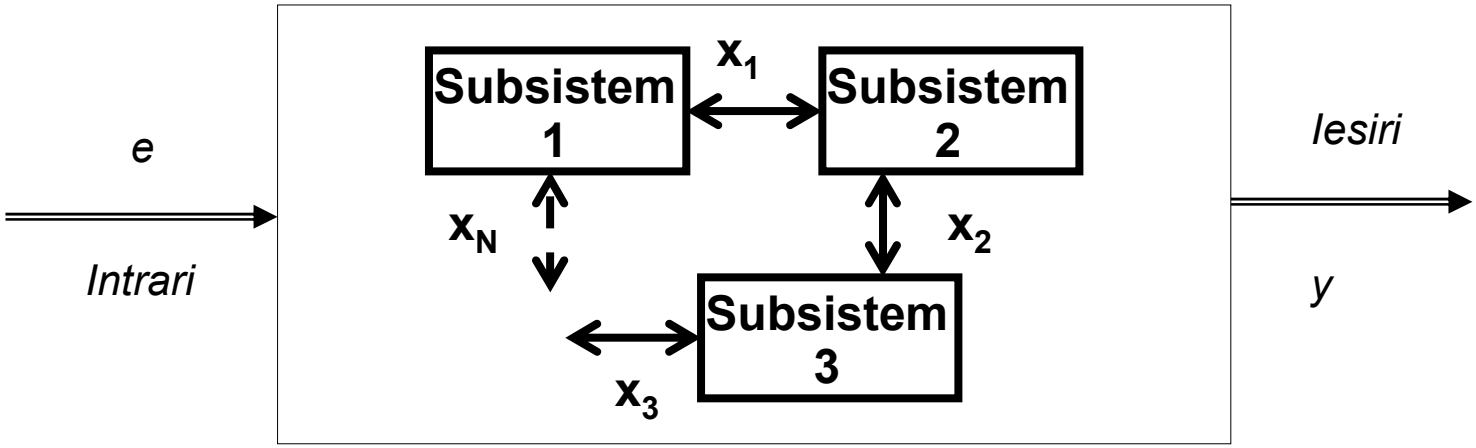
$$\sum_{m=0}^{M} a_m \cdot z^{-m} \cdot E(z) = \sum_{n=0}^{N} b_n \cdot z^{-n} \cdot Y(z)$$

- Se observa ca transformata Z transforma ecuatia cu diferente (dinamica, cu memorie) intr-o **ecuatie algebrica**;
- Din ecuatia algebrica se poate deduce **functia de transfer**:

$$H(z) = \frac{Y(z)}{E(z)} = \sum_{m=0}^{M} a_m \cdot z^{-m} \Big/ \sum_{n=0}^{N} b_n \cdot z^{-n}$$

Caracterizare intrare – **stare** - iesire

- Accent deosebit pe **starea interna** a sistemului si evolutia ei
- Studiul modului in care **dinamica starii** prelucreaza intrarea



- Ecuatiile de stare reflecta:
  - **Evolutia** dinamica a starii,
  - **Influenta intrarii** asupra starii si
  - Modul de **obtinere a iesirii** prin simplu calcul **algebric** din stare

Ecuatii de stare analogice

Forma generala a ecuatiilor de stare:

- Forma scalara:

$$\begin{cases} x_1'(t) = f_1\left(x_1(t), x_2(t), \ldots, x_N(t), e(t)\right) \\ x_2'(t) = f_2\left(x_1(t), x_2(t), \ldots, x_N(t), e(t)\right) \\ \vdots \\ x_N'(t) = f_N\left(x_1(t), x_2(t), \ldots, x_N(t), e(t)\right) \end{cases}$$

$$y(t) = g\left(x_1(t), x_2(t), \ldots, x_N(t), e(t)\right)$$

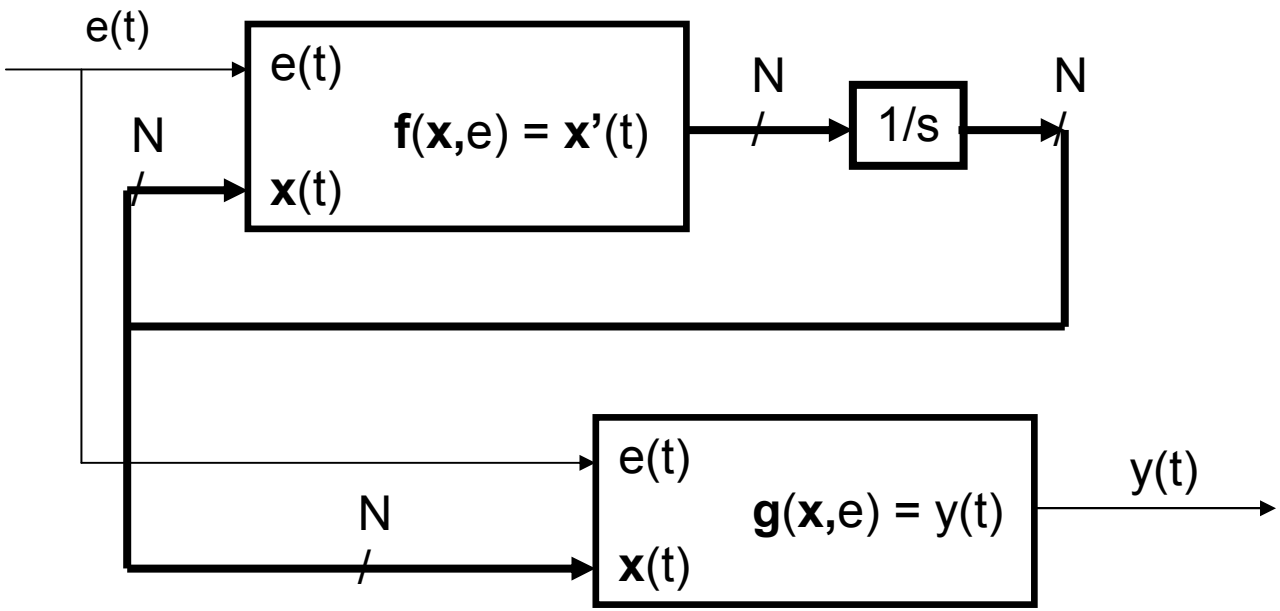
- Cu notatiile:

$$\begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_N(t) \end{bmatrix} \overset{not.}{=} \mathbf{x}(t) \qquad g\left(x_1, x_2, \ldots, x_N, e\right) \overset{not.}{=} g\left(\mathbf{x}, e\right) \qquad \begin{bmatrix} f_1(\mathbf{x}, e) \\ f_2(\mathbf{x}, e) \\ \vdots \\ f_N(\mathbf{x}, e) \end{bmatrix} \overset{not.}{=} \mathbf{f}(\mathbf{x}, e)$$

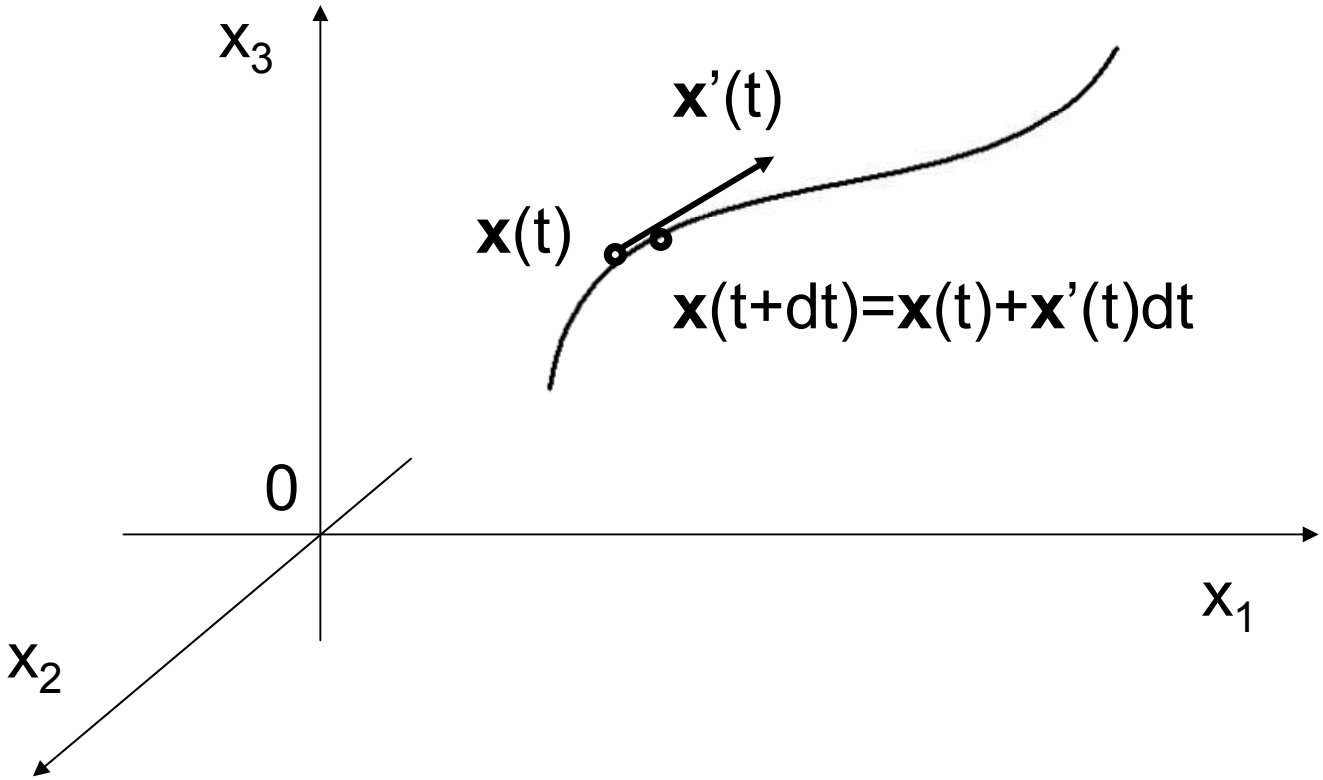
- Forma vectoriala:

$$\begin{cases} \mathbf{x}'(t) = \mathbf{f}\left(\mathbf{x}(t), e(t)\right) \\ y(t) = g\left(\mathbf{x}(t), e(t)\right) \end{cases}$$

Schema bloc de implementare analogica



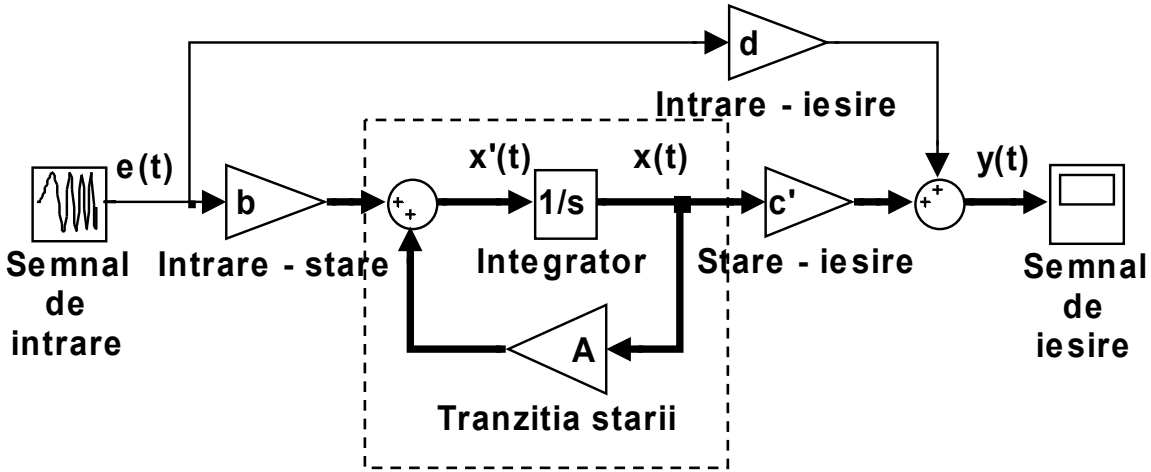
Interpretarea dinamica a ecuatiilor de stare



Cazul sistemelor liniare

- Ecuatiile de stare:

$$\begin{cases} \mathbf{x}'(t) = \mathbf{A}\cdot\mathbf{x}(t) + \mathbf{b}\cdot e(t) \\ y(t) = \mathbf{c}^{\mathrm{T}}\cdot\mathbf{x}(t) + d\cdot e(t) \end{cases}$$



Legatura intre ecuatiile de stare si functia de transfer

- Ecuatiile de stare in transformata Laplace:

$$\begin{cases} s\mathbf{X}(\mathrm{s}) = \mathbf{A}\cdot\mathbf{X}(\mathrm{s}) + \mathbf{b}\cdot\mathrm{E}(s) \\ Y(s) = \mathbf{c}^{\mathrm{T}}\cdot\mathbf{X}(\mathrm{s}) + d\cdot\mathrm{E}(s) \end{cases}$$

$$\left(s\mathbf{I}_N - \mathbf{A}\right)\cdot\mathbf{X}(\mathrm{s}) = \mathbf{b}\cdot\mathrm{E}(s) \qquad \mathbf{X}(\mathrm{s}) = \left(s\mathbf{I}_N - \mathbf{A}\right)^{-1}\cdot\mathbf{b}\cdot\mathrm{E}(s)$$

$$Y(s) = \mathbf{c}^{\mathrm{T}}\cdot\left(s\mathbf{I}_N - \mathbf{A}\right)^{-1}\cdot\mathbf{b}\cdot\mathrm{E}(s) + d\cdot\mathrm{E}(s)$$

$$H(s) = \frac{Y(s)}{\mathrm{E}(s)} = \frac{\mathbf{c}^{\mathrm{T}}\cdot\mathrm{Adj}\left(s\mathbf{I}_N - \mathbf{A}\right)^{T}\cdot\mathbf{b} + d\cdot\det\left(s\mathbf{I}_N - \mathbf{A}\right)}{\det\left(s\mathbf{I}_N - \mathbf{A}\right)}$$

Ecuatii de stare discrete

- Forma generala a ecuatiilor de stare discrete:
- Forma scalara:

$$\begin{cases} x_1[k+1] = f_1(x_1[k], x_2[k], \ldots, x_N[k], e[k]) \\ x_2[k+1] = f_2(x_1[k], x_2[k], \ldots, x_N[k], e[k]) \\ \vdots \\ x_N[k+1] = f_N(x_1[k], x_2[k], \ldots, x_N[k], e[k]) \end{cases}$$

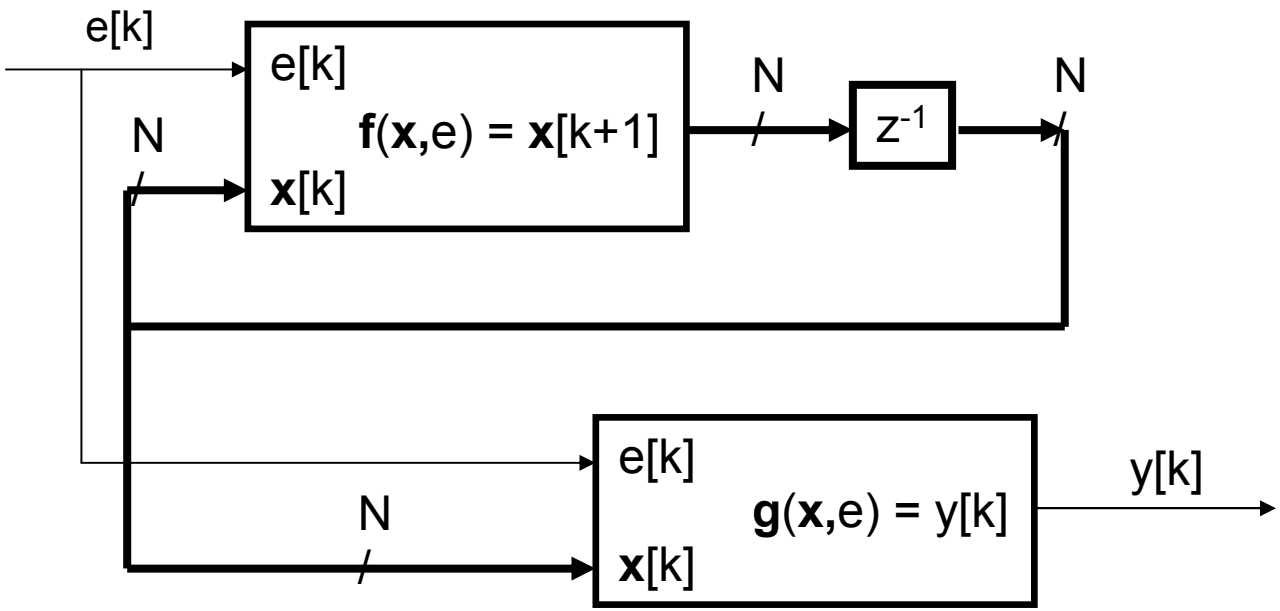
$$y[k] = g(x_1[k], x_2[k], \ldots, x_N[k], e[k])$$

- Cu notatiile: $\begin{bmatrix} x_1[k] \\ x_2[k] \\ \vdots \\ x_N[k] \end{bmatrix} \overset{not.}{=} \mathbf{x}[k]$ $\quad g(x_1, x_2, \ldots, x_N, e) \overset{not.}{=} g(\mathbf{x}, e)$ $\quad \begin{bmatrix} f_1(\mathbf{x}, e) \\ f_2(\mathbf{x}, e) \\ \vdots \\ f_N(\mathbf{x}, e) \end{bmatrix} \overset{not.}{=} \mathbf{f}(\mathbf{x}, e)$

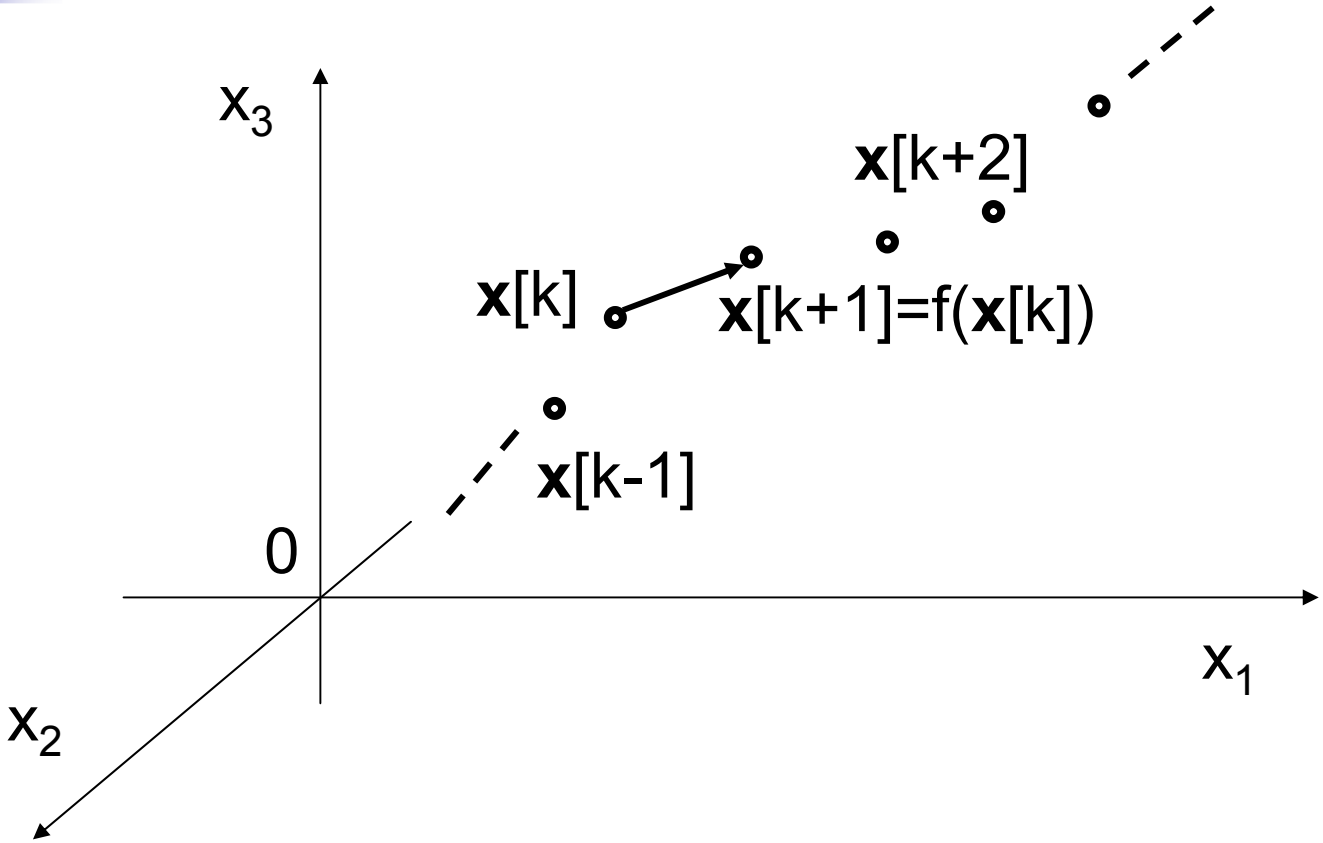
- Forma vectoriala:

$$\begin{cases} \mathbf{x}[k+1] = \mathbf{f}(\mathbf{x}[k], e[k]) \\ y[k] = g(\mathbf{x}[k], e[k]) \end{cases}$$

Schema bloc de implementare discreta



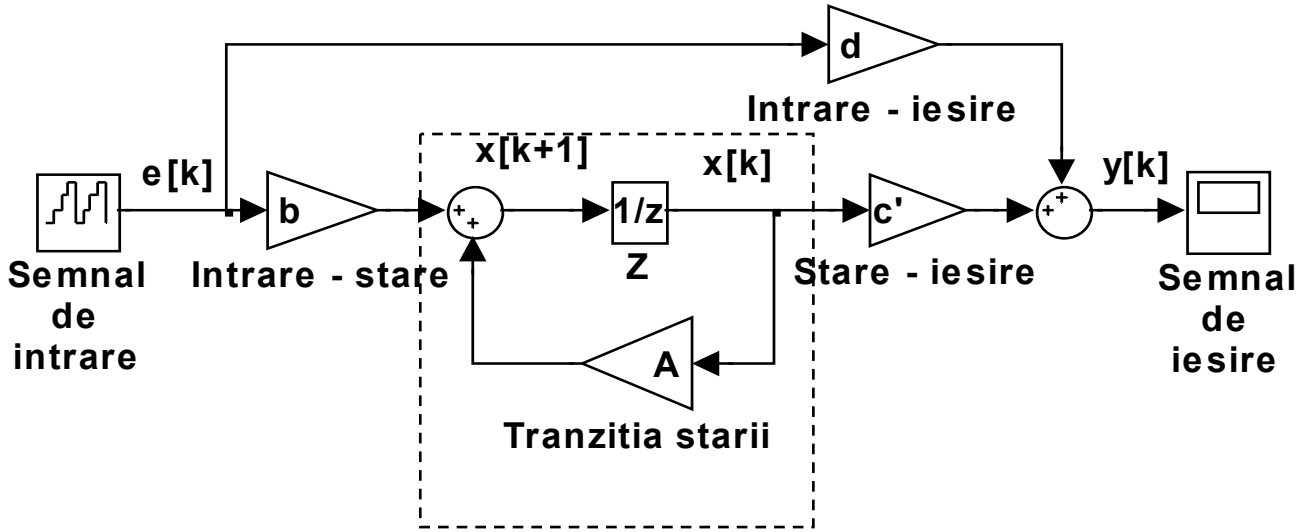
Interpretarea dinamica a ecuatiilor de stare discrete



Cazul sistemelor liniare discrete

- Ecuatiile de stare:

$$\begin{cases} \mathbf{x}[k+1] = \mathbf{A} \cdot \mathbf{x}[k] + \mathbf{b} \cdot e[k] \\ y[k] = \mathbf{c}^{\mathrm{T}} \cdot \mathbf{x}[k] + d \cdot e[k] \end{cases}$$



Legatura intre ecuatiile de stare si functia de transfer

- Ecuatiile de stare in transformata Z:

$$\begin{cases} z\mathbf{X}(\mathrm{z}) = \mathbf{A}\cdot\mathbf{X}(\mathrm{z}) + \mathbf{b}\cdot\mathrm{E}(z) \\ Y(z) = \mathbf{c}^{\mathrm{T}}\cdot\mathbf{X}(\mathrm{z}) + d\cdot\mathrm{E}(z) \end{cases}$$

$$\left(z\mathbf{I}_N - \mathbf{A}\right)\cdot\mathbf{X}(\mathrm{z}) = \mathbf{b}\cdot\mathrm{E}(z) \qquad \mathbf{X}(\mathrm{z}) = \left(z\mathbf{I}_N - \mathbf{A}\right)^{-1}\cdot\mathbf{b}\cdot\mathrm{E}(z)$$

$$Y(z) = \mathbf{c}^{\mathrm{T}}\cdot\left(z\mathbf{I}_N - \mathbf{A}\right)^{-1}\cdot\mathbf{b}\cdot\mathrm{E}(z) + d\cdot\mathrm{E}(z)$$

$$H(z) = \frac{Y(z)}{\mathrm{E}(z)} = \frac{\mathbf{c}^{\mathrm{T}}\cdot\mathrm{Adj}\left(z\mathbf{I}_N - \mathbf{A}\right)^{T}\cdot\mathbf{b} + d\cdot\det\left(z\mathbf{I}_N - \mathbf{A}\right)}{\det\left(z\mathbf{I}_N - \mathbf{A}\right)}$$

Concluzii

- **Ecuatiile de stare** sunt ca mai potrivita caracterizare matematica a sistemelor si circuitelor **neliniare**
- Caracterizarea **intrare-stare-iesire** este mai detaliata:
  - Separa **dinamca interna** a circuitului de conexiunile de intrare si iesire
  - Permite calculul raspunsului la semnal de intrare, dar si la **starea initiala**
  - Constituie baza pentru deducerea **schemei bloc** a sistemului neliniar studiat, pentru **simulare** si **realizare**
  - Ofera interpretarea intuitiva a dinamicii sistemului
- Ecuatiile de stare pentru sistemele liniare sunt particularizari ale cazului mai general, neliniar
- Din acestea se poate deduce functia de transfer, permitand evidentierea influentei matricei **A** asupra dinamicii interne si a conexiunilor de intrare si iesire (**b**, **c**, d) asupra tipului de selectivitate (TJ, TS, TB,…)