Using Different Bias Current Sources for Controlling Turing Patterns

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Abstract

For several years some phenomena were reported in two-grid coupled arrays made with Chua's cell. One of them is the formation of Turing patterns. Until now, there were two ways for controlling Turing patterns formation in CNNs: using the dispersion curve and respectively using different initial conditions [1]. This work presents a new way of controlling Turing pattern formation in CNNs: by using different bias current source of the Chua's cell.

1 Introduction

The two-grid-coupled network is described by a system of non-linear differential equations.

$$\begin{cases} \dfrac{du_i(t)}{dt} = \gamma f(h(u_i), v_i) \\ \dfrac{dv_i(t)}{dt} = \gamma g(u_i, v_i) \end{cases}, \quad i = 0 \ldots M-1$$

where h(u) has the form:

$$h(u) = \begin{cases} m_2 u + (m_2 - m_0) + \varepsilon, & u < -1 \\ m_0 u + \varepsilon, & -1 \le u \le 1 \\ m_1 u + (m_0 - m_1) + \varepsilon, & u > 1 \end{cases}$$

The cell will operate in the middle linear part and thus h(u) equals $m_0 u+\varepsilon$.

2 The decoupling technique

The main technique for solving this system consists in de-coupling the state variables. They can be de-coupled by writing the solution as a function of a weighted sum of spatial eigen-functions, in the form, depending on

$$\begin{cases} u_i(t) = \sum_{m=0}^{M-1} \phi(m,i)\, \hat{u}_m(t) \\ v_i(t) = \sum_{m=0}^{M-1} \phi(m,i)\, \hat{v}_m(t) \end{cases}, \quad i = 0 \ldots M-1$$

the boundary type the basis of DCT (zero-flux boundary) or FFT (periodic boundary) [2]. If we consider an non-homogeneous bias current source with a different value for each cell, then these values can be viewed as a spatial input signal.

The linearized system of equations around the origin can be written as bellow, where the ∇ denotes 1D discrete laplacean.

Furthermore, we can decompose the spatial signal generated by biases taking into account the orthogonal

$$\begin{bmatrix} u_i(t) \\ v_i(t) \end{bmatrix}' = \gamma \begin{bmatrix} f_u & f_v \\ g_u & g_v \end{bmatrix} \begin{bmatrix} u_i(t) \\ v_i(t) \end{bmatrix} + \gamma \begin{bmatrix} \varepsilon \\ 0 \end{bmatrix} + \begin{bmatrix} D_u & 0 \\ 0 & D_v \end{bmatrix} \begin{bmatrix} \nabla^2 u_i \\ \nabla^2 v_i \end{bmatrix}$$

basis. The orthogonal functions are eigen-functions for the laplacean operator [3]. Considering that, the system of equations is given above, where k_m^2 has, for

$$\begin{bmatrix} \sum_{m=0}^{M-1} \phi(m,i)\, \hat{u}_m(t) \\ \sum_{m=0}^{M-1} \phi(m,i)\, \hat{v}_m(t) \end{bmatrix}' = \gamma \begin{bmatrix} f_u & f_v \\ g_u & g_v \end{bmatrix} \begin{bmatrix} \sum_{m=0}^{M-1} \phi(m,i)\, \hat{u}_m(t) \\ \sum_{m=0}^{M-1} \phi(m,i)\, \hat{v}_m(t) \end{bmatrix} +$$

$$\gamma \begin{bmatrix} \sum_{m=0}^{M-1} \phi(m,i)\, \hat{\varepsilon}_m \\ 0 \end{bmatrix} + \begin{bmatrix} D_u & 0 \\ 0 & D_v \end{bmatrix} \begin{bmatrix} \sum_{m=0}^{M-1} -k_m^2 \phi(m,i)\, \hat{u}_m(t) \\ \sum_{m=0}^{M-1} -k_m^2 \phi(m,i)\, \hat{v}_m(t) \end{bmatrix}$$

example, in the case of choosing ϕ(m,i) as members of DCT basis the form described bellow, (the case of

$$k_m^2 = 4 \sin^2\left(\frac{m\pi}{2M}\right)$$

choosing ϕ(m,i) as members of FFT basis was previously detailed and the calculus is perfectly identical).

We multiply the expression by ϕ(n,i), replace n by m and obtain the expression:

$$\begin{bmatrix} \hat{u}_m(t) \\ \hat{v}_m(t) \end{bmatrix}' = \gamma \begin{bmatrix} f_u & f_v \\ g_u & g_v \end{bmatrix} \begin{bmatrix} \hat{u}_m(t) \\ \hat{v}_m(t) \end{bmatrix} +$$

$$\gamma \begin{bmatrix} \hat{\varepsilon}_m \\ 0 \end{bmatrix} - k_m^2 \begin{bmatrix} D_u & 0 \\ 0 & D_v \end{bmatrix} \begin{bmatrix} \hat{u}_m(t) \\ \hat{v}_m(t) \end{bmatrix}$$

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$$\begin{cases} \hat{u}_m(t) = a_m(\hat{\varepsilon}_m, \hat{u}_m(0), \hat{v}_m(0)) e^{\lambda_{m1} t} + b_m(\hat{\varepsilon}_m, \hat{u}_m(0), \hat{v}_m(0)) e^{\lambda_{m2} t} + f_1(\hat{\varepsilon}_m) \\ \hat{v}_m(t) = c_m(\hat{\varepsilon}_m, \hat{u}_m(0), \hat{v}_m(0)) e^{\lambda_{m1} t} + d_m(\hat{\varepsilon}_m, \hat{u}_m(0), \hat{v}_m(0)) e^{\lambda_{m2} t} + f_2(\hat{\varepsilon}_m) \end{cases}$$

This is an non-homogeneous linear differential equation system. Its solution is displayed above.

3 How the spatial signal influences final pattern?

The answer to this question must be given taking into account the therms that contain spectrum elements of spatial signal ε. First we have to determine the shape of $f_1(\hat{\varepsilon}_m)$ and $f_2(\hat{\varepsilon}_m)$, respectively. We impose the condition that the solution must satisfy the system of equations for all moments of time. We derive the following expressions for $f_1(\hat{\varepsilon}_m)$ and $f_2(\hat{\varepsilon}_m)$:

So, the spectral element of the spatial signal generated by bias current sources is "amplified" by the terms detailed above. In addition, a_m, b_m, c_m and d_m depend on the same spectral element. Consequently, the final

$$\begin{cases} f_1(\hat{\varepsilon}_m) = \dfrac{-\gamma(\gamma g_v - k_m^2 D_v)}{(\gamma f_u - k_m^2 D_u)(\gamma g_v - k_m^2 D_v) - \gamma^2 f_v g_u} \hat{\varepsilon}_m \\ f_2(\hat{\varepsilon}_m) = \dfrac{\gamma^2 g_u}{(\gamma f_u - k_m^2 D_u)(\gamma g_v - k_m^2 D_v) - \gamma^2 f_v g_u} \hat{\varepsilon}_m \end{cases}$$

pattern will be influenced by the bias current sources in two ways:

- by means the f_1 and f_2 terms (linearly amplifies the corresponding mode);
- by means the exponential therms, which have coefficients that depend on $\hat{\varepsilon}_m$ and on the initial conditions on the capacitors. If the mode is located within the unstable band of the dispersion curve, then the respective mode will develop (it corresponds to an eigen-value with positive real part). If the mode is located "outside" the dispersion curve, then it will decrease to zero (it corresponds to an eigen-value with negative real part).

The solution is:

$$\begin{cases} u_i(t) = \sum_{m=0}^{M-1} (a_m(\hat{\varepsilon}_m, \hat{u}_m(0), \hat{v}_m(0)) e^{\lambda_{m1} t} + b_m(\hat{\varepsilon}_m, \hat{u}_m(0), \hat{v}_m(0)) e^{\lambda_{m2} t} + A_u^m \hat{\varepsilon}_m) \phi(m,i) \\ v_i(t) = \sum_{m=0}^{M-1} (c_m(\hat{\varepsilon}_m, \hat{u}_m(0), \hat{v}_m(0)) e^{\lambda_{m1} t} + d_m(\hat{\varepsilon}_m, \hat{u}_m(0), \hat{v}_m(0)) e^{\lambda_{m2} t} + A_v^m \hat{\varepsilon}_m) \phi(m,i) \end{cases}, \quad i = 0 \ldots M-1$$

The solutions can be put into the form:

$$\begin{cases} u_i(t) = \sum_{m=0}^{M-1} (a_m e^{\lambda_{m1} t} + b_m e^{\lambda_{m2} t}) \phi(m,i) + \sum_{m=0}^{M-1} A_u^m \hat{\varepsilon}_m \phi(m,i) \\ v_i(t) = \sum_{m=0}^{M-1} (c_m e^{\lambda_{m1} t} + d_m e^{\lambda_{m2} t}) \phi(m,i) + \sum_{m=0}^{M-1} A_v^m \hat{\varepsilon}_m \phi(m,i) \end{cases}, \quad i = 0 \ldots M-1$$

The behavior of this system will follow the rules:

- modes "inside" the dispersion curve will be "favored" by the network. Due to the fact that the coefficients a_m, b_m, c_m and d_m depend on spatial signal generated by bias current sources, the modes from the spatial signal that are located inside the dispersion curve will increase, too.;
- all modes of the spatial signal made by bias current sources will be "amplified" with an A^m_u.

4 Computer simulations

In this part some significant simulation results are presented. First of all, we verify the first rule mentioned above. We make that by choosing a network whose dispersion curve is located under the x-axis and consequently there are not any modes located "inside" the dispersion curve. A set of parameters which satisfy these rules is: fu=0.1, fv=-1, gu=0.1, gv=-0.2, Du=1, Dv=10, γ=1, M=30. We use a signal generated by bias current sources that has only one spatial mode in its spectrum: mode 5. The amplitude of the mode will be initially 0.1. According to the first rule mentioned above, the mode 5 will be amplified with 4.9.

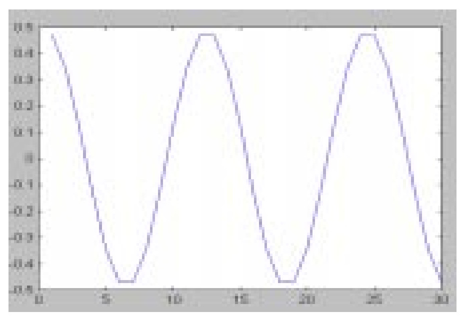

Fig. 1: Final pattern

The final "pattern" is represented in the Fig. 1.

The second simulation presents a phenomenon related to the second rule described above. The dispersion curve will have inside two modes: 4 and 5.

The parameters are the same, except Dv=50 and γ=5. The corresponding dispersion curve is represented in Fig. 2.

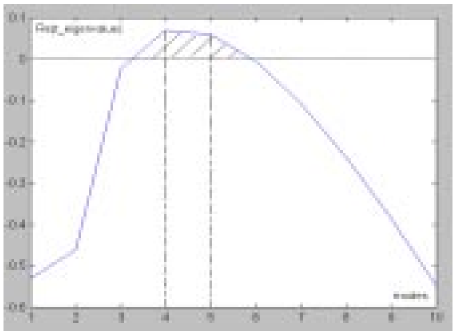

Fig 2: Dispersion curve

We seed the network with spatial mode 4 of amplitude 0.1 on the u-nodes capacitors and the spatial signal made by the bias sources will have only the spatial mode 2 of amplitude 0.01 in its spectrum. The final pattern is represented below, in Fig. 3.

Fig. 3: The final pattern

The corresponding spectrum of final pattern is displayed in Fig. 4.

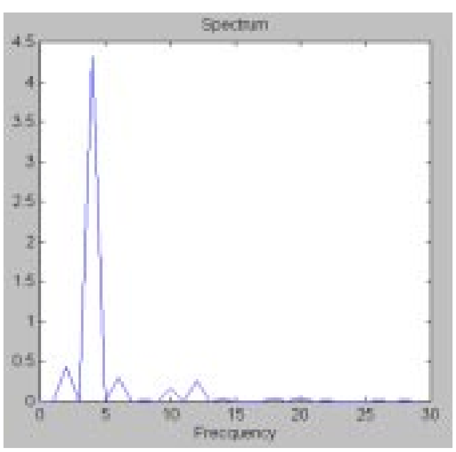

Fig. 4: The spectrum of final pattern

If one increases the amplitude value of spatial mode a different pattern is obtained. Let us set it to 0.05. Final pattern is represented below, in Fig. 5.

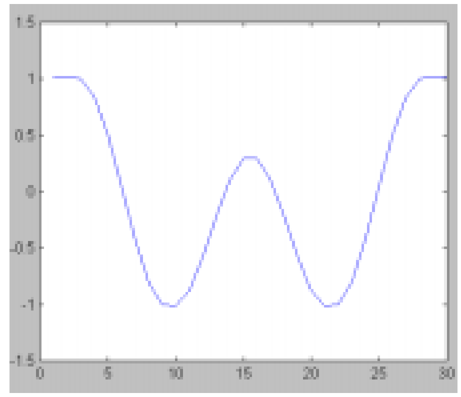

Fig. 5: Final Pattern

The spectrum is also represented in Fig. 6:

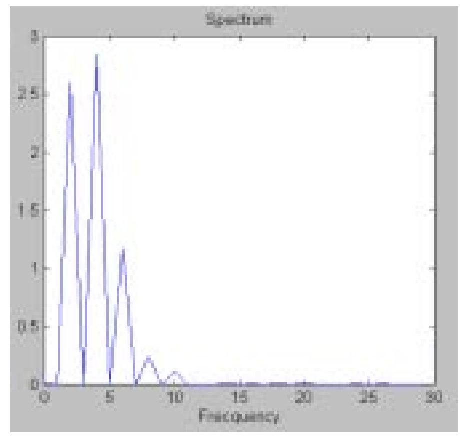

Fig. 6: The spectrum of final pattern

Let us notice that mode 2 has “grown” five times, corresponding to the growth of the “initial condition” imposed for the spatial signal source generated by bias current sources from 0.01 to 0.05. If the amplitude of that mode is increased more (0.1) then the final pattern will look like in the figure below:

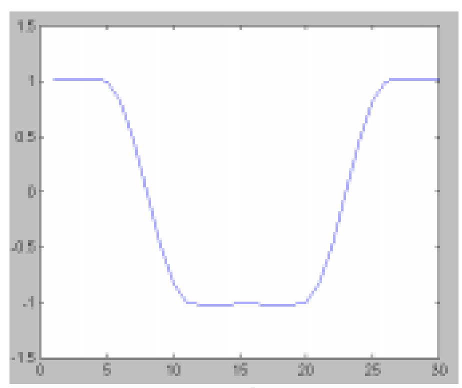

Fig. 7: Final pattern

Spectrum analysis allows us to make an interesting observation: the spatial signal on the state variables (capacitors) corresponding to the "classical" initial conditions has been disappeared due to the “growth” of the mode introduced through the spatial signal generated by bias current sources:

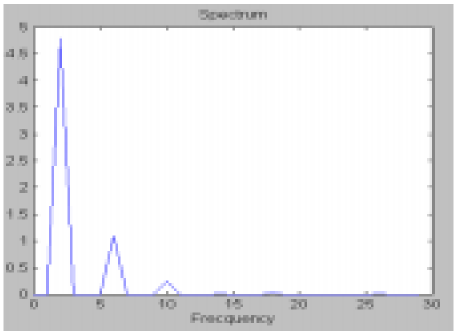

Fig. 8: The spectrum of final pattern

It can be easily noticed that there is only the mode 2 in the spectrum and the harmonics generated by the non-linearity.

The last experiment we do corresponds to the following situation: we seed one mode that is located "inside" the dispersion curve on the capacitors and one mode that is located "inside" the dispersion curve on the bias current sources of each cell. The same parameters for the network will be used, except the parameter γ which will be increased. By this increase, the dispersion curve will move significantly to the right. The dispersion curve is represented in Fig. 9.

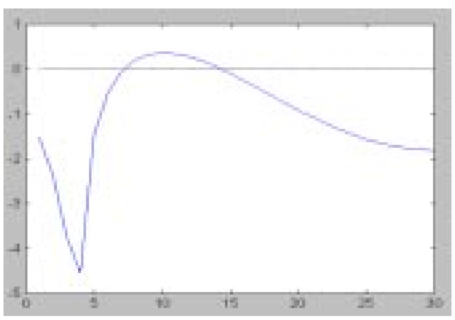
Fig. 9: Dispersion curve

The modes located "inside" the dispersion curve are displayed in the following table:

Table 1: Real part of the positive eigenvalues

	Modes	Real(eigenvalue)
Modes inside the dispersion curve	8	0.2061
	9	0.3331
	10	0.3713
	11	0.3471
	12	0.2777
	13	0.1752
	14	0.0483

Mode 12 is seeded on the capacitors. It has an amplitude of 0.1. Mode 13 is seeded on the bias current sources with an amplitude of 0.1.

The final pattern obtained is represented in Fig. 9.

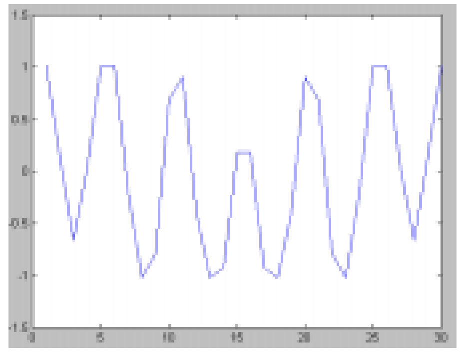
Fig. 9: Final pattern

Its spectrum is represented below, in the Fig. 10:

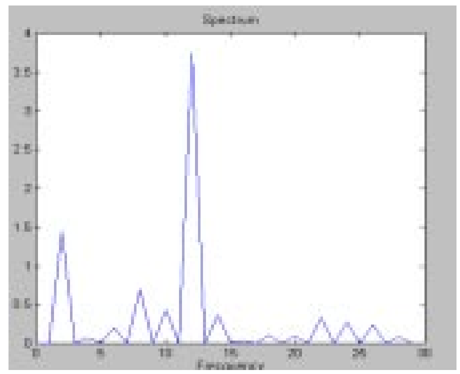
Fig. 10: The spectrum of final pattern

5 Conclusions

In the presented paper a new pattern control type is studied. Beside the control using the dispersion curve and the initial conditions imposed on capacitors, we propose another type of control using the values of the bias current source of each cell. The implementation cost should not increase significantly compared to the case when all the current sources have one fixed value.

References

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