On Pattern Formation in a Class of Cellular Neural Networks

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Abstract - **The dynamics of a class of Cellular Neural Networks (CNN's) related to pattern formation is investigated in the central linear part using the decoupling technique based on Discrete Spatial Fourier Transform. The influence of the cell order and template neighborhood is discussed as well**

I. Introduction

CNN's - homogeneous arrays of identical cells identically coupled - have been intensely investigated in the last decade for their applications in fast image processing [[1-9]. The standard CNN [1] consists of an input source, an RC parallel circuit, a biasing source and a nonlinear controlled source, which converts the state of the cell into the output. The state of each cell is determined by the outputs and inputs of the neighboring cells - within prescribed radius - through controlled sources. The image to be processed is introduced through the input and/or the state of the cells, each cell of the array corresponding to a pixel of the image.

An interesting phenomenon [10-17], which has been shown to appear in CNN's, is pattern formation.In this communication, the dynamics in the central linear part of a class of 1D CNN's will be investigated emphasizing the influence of the cell order and the template neighbourhood radius. The M CNN cells are supposed to be nonlinear (piecewise linear) dynamic one-ports, which for the central linear part behave linearly as a one-port admittance Y(s) as shown in Fig. 1. When the system is unstable, the evolution of the CNN is supposed to be stopped before some nonlinear mechanism would limit the growth of the signals.

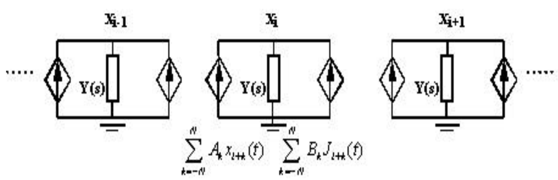


Fig. 1. 1D CNN model for the central linear part

The CNN is described, by the following set of differential equations:

$$Y(s)x_i(t) = \sum_{k=-N}^{N} A_k x_{i+k}(t) + \sum_{k=-N}^{N} B_k J_{i+k}(t) \qquad (1)$$

where s=d/dt, N is the neighborhood radius and A_k and B_k characterize the connection with neighboring cell outputs and the input sources Ji(t), respectively. The two templates are supposed to be of the same dimensions but this is not a restriction as part of the coefficients can be zero.

II. Solving the equations

Using the decoupling technique, i.e., the change of variables.

$$x_i(t) = \sum_{m=0}^{M-1} \Phi_M(i,m)\hat{x}_m(t)$$
$$J_i(t) = \sum_{m=0}^{M-1} \Phi_M(i,m)\hat{J}_m(t) \qquad (2)$$

i=0,1,…,M-1, with eigenfunctions Φ_M(i,m) of the form (ring BC's)

$$\Phi_M(i,m) = e^{j\frac{2p}{M}mi} \qquad (3)$$

in the general case, the action of the A-template (similar relations are valid for the B-template) gives:

$$\sum_{k=-N}^{N} A_k e^{j\frac{2p}{M}m(i+k)} = K_A(m) e^{j\frac{2p}{M}mi} \qquad (4)$$

where:

$$K_A = \sum_{k=-N}^{N} A_k e^{j\frac{2p}{M}mk} \qquad (5)$$

The positions of the eigenvalues $K_A(m)$ for several templates corresponding to first and second order neighborhoods are given in Fig. 2.

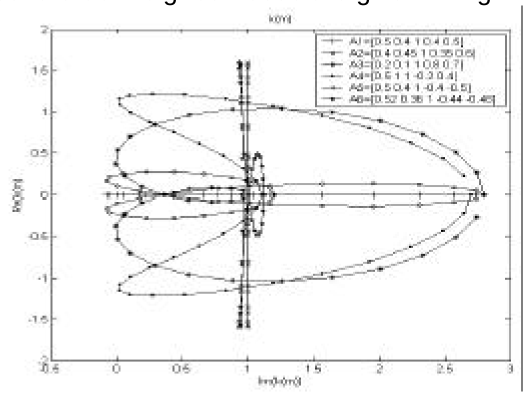


Fig.2. Eigenvalues positions in the complex plane for several second order templates.

With the above change of variables and taking successively the scalar product with $F_M(i,m)$ of both sides, the equations decouple. Thus, each spatial mode, m, is characterized by the differential equation

$$Y(s)\hat{x}_m(t) = K_A(m)\hat{x}_m(t) + K_B(m)\hat{J}_m(t \qquad (6)$$

and the transfer function

$$H_m(s) = \frac{K_B(m)}{\dfrac{Q(s)}{P(s)} - K_A(m)} \qquad (7)$$

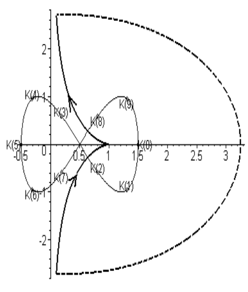
where Y(s)=Q(s)/P(s), P(s) and Q(s) being polynomials in the variable s.

The decoupling technique allows the study of the CNN dynamics based on the time evolution of each spatial mode. The stability and dynamics of each spatial mode are determined by the zeros of

$$\frac{Q(s)}{P(s)} - K_A(m) \qquad (8)$$

.Taking advantage of the special form of this equation, the dynamics of the CNN can be studied using techniques from feedback theory like the Nyquist criterion adapted for complex amplifications as, in the general case of asymmetric templates, ($A_i \neq A_{-i}$) $K_A(m)$ is complex and satisfies the condition $K_A(m)=K^*_A(M-m)$.

Indeed, the stability depends on the relative position of $K_A(m)$ with respect to the hodograph of Q(s)/P(s) for s belonging to the Nyquist contour. The stability of the modes is determined by the number of times $K_A(m)$ and $K^*_A(m)$. are circled by the hodograph of Y(s): the shape of the hodograph of Y(s) gives the influence of the cell on the dynamics while the template influence is given by the position of the eigenvalues with respect to the hodograph. An example is given in the figures below for a CNN made of cells with admittance: Y(s)=(s2+s+1)/(s+1), A=[0.5,0.5,0.5,0.5,-0.5], J(t)=0. For random initial conditions, the time evolution shows that the winning mode is m=1 which corresponds to complex conjugated temporal eigenvalues, so that the dynamic is oscillatory.



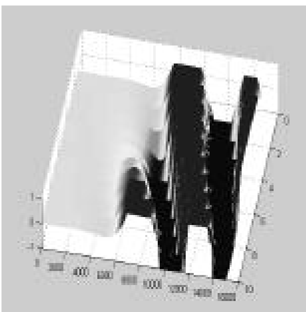


Fig. 3 a. Hodograph of Y(s) and spatial eigenvalues b. Output evolution for random IC

In the following we will consider first-order cell CNN's for which the Nyquist hodograph corresponding to the imaginary axis is a vertical line as Q(s)/P(s)=s. Thus, the roots of the characteristic polynomial will be identical with the spatial eigenvalues, $?_m=K_A(m)$. When M=2N+1, i.e., any cell is coupled with all other cells,

$$l_m = \sum_{k=-N}^{N} A_k e^{j\frac{2p}{2N+1}mk} \qquad (9)$$

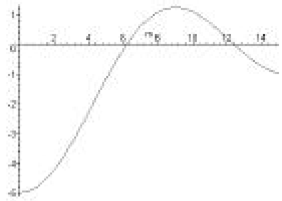
the template coefficients and the temporal eigenvalues are Discrete Fourier Series pairs. Based on this observation, the design of a template which produces a prescribed configuration of the spatial modes temporal eigenvalues resides in solving an inverse DFS using perhaps the FFT algorithm. On the other hand, such large templates are impractical for a possible implementation.

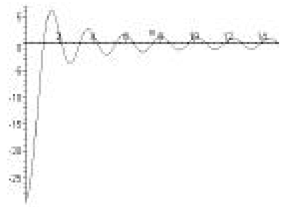
Reducing the template dimension leads to the modification of the initial temporal eigenvalues positions. Formally, using a smaller template (M > 2N+1) is equivalent to the use of a 2N+1 constant window, which corresponds to a convolution in the eigenvalues domain with the function:

$$l_m = \frac{\sin\left(\dfrac{mp\,(2N+1)}{M}\right)}{\sin\left(\dfrac{mp}{M}\right)} \qquad (10)$$

which represents the eigenvalues positions for a constant negative window-type template.

Two particular cases are presented below for a CNN with M=31. The figures represent the envelope of the real part of the eigenvalues (which, in this case are real) and their values.





m=0	-5	m=8	1.10
m=1	-4.80	m=9	1.25
m=2	-4.21	m=10	1.10
m=3	-3.34	m=11	0.72
m=4	-2.28	m=12	0.21
m=5	-1.18	m=13	-0.31
m=6	-0.18	m=14	-0.73
m=7	0.60	m=15	-0.97

m=0	-29	m=8	1.38
m=1	-1.99	m=9	-1.22
m=2	1.96	m=10	1.06
m=3	-1.91	m=11	-0.88
m=4	1.84	m=12	0.69
m=5	-1.75	m=13	-0.50
m=6	1.64	m=14	0.30
m=7	-1.52	m=15	-0.10

Fig. 4 Envelope and eigenvalues for N=2 and N=14

Using the constant window template corresponds to a kind of comb filter: successive spatial modes will have alternatively stable and unstable behaviour, according to the shape of transform of the constant window. The modes with positive real part will develop while those with negative real part will attenuate with time

The mechanism is illustrated in the figure below:

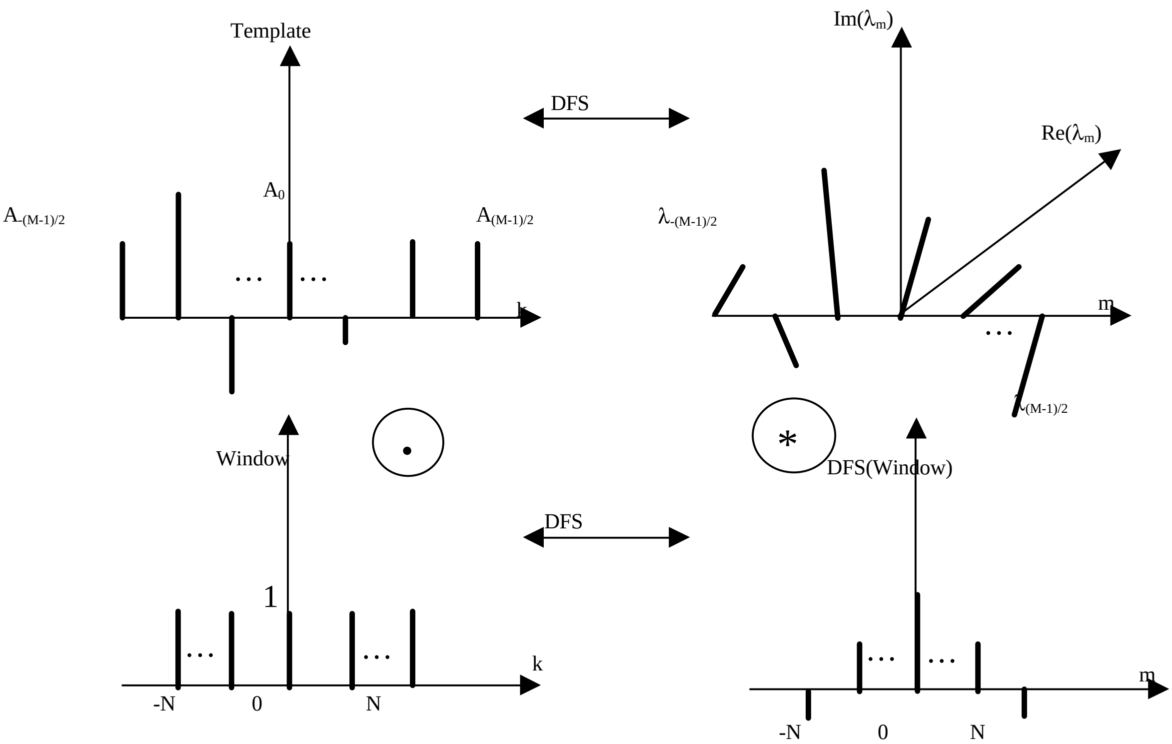
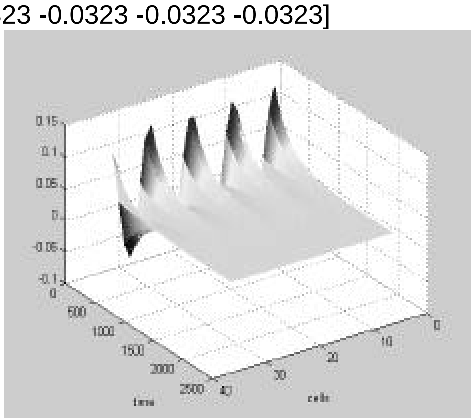
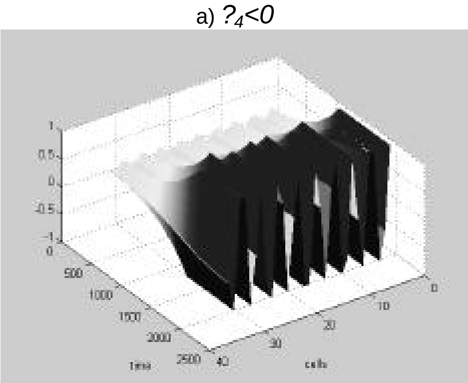


Fig. 4. Reducing the template neighborhood

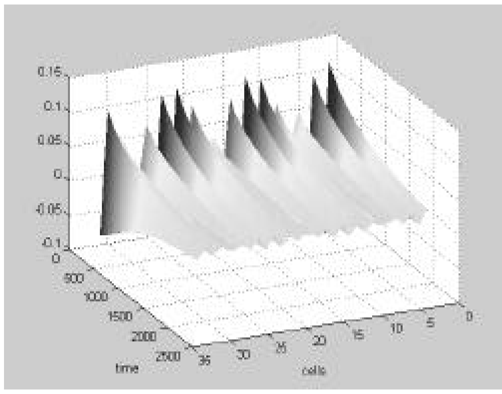
The simulation results are given below with N=2 and different types of initial conditions A=[-0.0323 -0.0323 -0.0323 -0.0323 -0.0323 -0.0323]



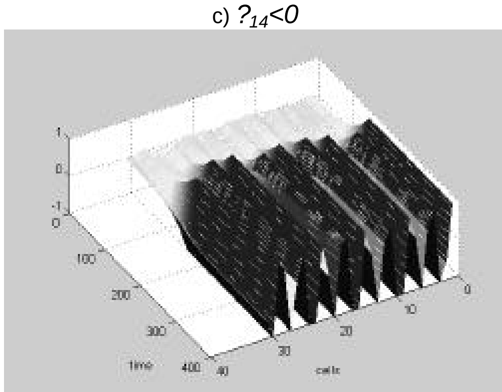
a) $?_4<0$



b) $?_8>0$



c) $?_{14}<0$



d) Random initial conditions

Fig. 5. Time evolution for several spatial modes

III. Concluding remarks

In this work, several analytical results concerning the dynamics of CNN and pattern formation have been presented. They are based on the decoupling technique, which, although intrinsically linear, give significant insight for pattern formation. The method has been combined with the Nyquist techniques and puts into evidence the dynamics of the CNN with respect to the Fourier components of the input and initial conditions for ring boundary.

IV. Acknowledgements

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