Cell and Template Order Influence on CNN Behavior - A Comparative Study

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***Abstract* - In this communication comparatives study of the dynamics of first order cell - second order neighborhood and second order cell – first order neighborhood CNN’s is presented.**

The comparison is made in terms of the dispersion curve, which is based on the decoupling technique and valid for the central linear part of the piecewise linear cell characteristics.

1 Introduction

Since their invention [1-5] many significant results regarding the CNN behavior have been obtained. One of the methods for patterns studying and filtering capabilities makes use of the decoupling technique, which is valid for dynamics, restricted to the central linear part of the cell characteristics. This method is fundamentally linear and has been applied for the study of the linear filtering properties of the first order cell and second order neighborhood template as well as for the pattern formation in second order cell first order neighborhood template. The aim of this communication is to make a comparison between the two kind of CNN;s, using the decoupling technique and the dispersion curves [6]. The main idea of the above technique is to use a change of variable based on a system of orthogonal discrete spatial functions (chosen according to the boundary conditions). The change of variable leads to a system of pairs of decoupled linear differential equations having as variables the spatial spectrum of the node voltages with respect to the orthogonal spatial functions. The dispersion curves represent the dependence of the real part of the system temporal eigenvalues vs. either spatial eigenvalues or vs. spatial modes (frequencies).

2 First order cell – first order neighborhood CNN's

In the following we consider a CNN made of first order cells and first order neighborhood template. The equations, valid in the central linear part, are:

$$\frac{du_i(t)}{dt} = -u_i + O_{1D}(u_i) + \varepsilon_i, \qquad i = 0..M \qquad (1)$$

where the operator $O_{1D}(u_i)$ depends on the template coefficients A, B and C:

$$O_{1D}(u_i) = Bu_{i-1} + Cu_{i+1} + Au_i \qquad (2)$$

and ε_i stands for the biasing.

For a symmetric template (C=B), the eigenvalues of the spatial operator O_{1D} associated to the eigenfunctions

$$\Phi_M(\omega(m), \varphi(m), i) = e^{j(\omega(m)i + \varphi(m))} \qquad (3)$$

are real numbers:

$$\boxed{K_{1D} = A + 2B\cos(\omega(m))} \qquad (4)$$

which does not depend on the phase φ(m).

In the following, only symmetric templates will be considered.

Looking for a solution of the form:

$$u_i = \sum_{m=0}^{M-1} \Phi_M(m,i)\,\hat{u}_m \qquad i = 0..M-1 \qquad (5)$$

where Φ_M(m,i) are the spatial orthogonal functions and using the same technique as in [6] we obtain the following set of first order decoupled equations:

$$\frac{d\,\hat{u}_m(t)}{dt} = \left(-1 + K_{1D}\right)\hat{u}_m + \hat{\varepsilon}_m, \quad i = 0..M \qquad (6)$$

where $\hat{\varepsilon}_m$ is the spectrum of the biasing signal with respect to the orthogonal functions.

The above equations have (real) temporal eigenvalues. Their dependence on the spatial eigenvalues (dispersion curve) is an affine relationship:

$$\mathrm{Re}(\lambda(K_{1D})) = K_{1D} - 1 \qquad (7)$$

(where the operator that takes the real part is superfluous).

3. Second order cell – first order neighborhood CNN's

In the following we consider two-port cells connected by means of two identical first order neighborhood templates.

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In the central linear part, the CNN is described by the following set of equations

$$\begin{cases} \dfrac{du_i(t)}{dt} = \gamma(f_u u_i + f_v v_i) + D_u O_{1D}(u_i) \qquad i = 0..M-1 \\ \dfrac{dv_i(t)}{dt} = \gamma(g_u u_i + g_v v_i) + D_v O_{1D}(v_i) \end{cases} \tag{8}$$

where O_{1D} has the same significance as before.

Using the decoupling technique, by means of the change of variable:

$$\begin{cases} u_i = \sum_{m=0}^{M-1} \Phi_M(m,i)\hat{u}_m \qquad i = 0..M-1 \\ v_i = \sum_{m=0}^{M-1} \Phi_M(m,i)\hat{v}_m \end{cases} \tag{9}$$

It turns out that the spatial eigenvalues are the same as (4) and the dispersion curve for a CNN with different templates for the u and v layers is given by the equation below:

$$\begin{aligned} &\mathrm{Re}(\lambda_{1,2}(K_{1D}, K_{1D}^{'})) = \\ &\mathrm{Re}\{\gamma\frac{f_u + g_v}{2} + \frac{D_u K_{1D} + D_v K_{1D}^{'}}{2} \\ &\pm\sqrt{\left(\gamma\frac{(g_v - f_u)}{2} - \frac{(D_u K_{1D} - D_v K_{1D}^{'})}{2}\right)^2 + \gamma^2 f_v g_u}\} \end{aligned} \tag{10}$$

For identical and symmetrical templates for the two layers, the spatial eigenvalues K_{1D} and K_{1D}' have the same value which is real. The dispersion curves for such a case are represented in Figure 1.

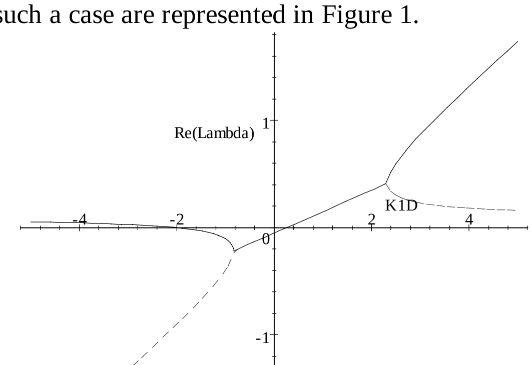

Figure 1 : The real part of both temporal eigenvalues versus spatial eigenvalue (same for each layer)

The continuous curve represents the locus of the real part of the solution with the "+" sign and the dotted curve represents the locus of the real part of the solution with "–" sign.

The curves have three zones:

- the left-zone, which corresponds to distinct real roots;
- the middle-zone (complex-conjugated roots);
- the right zone, (distinct real roots).

From the above dispersion curve one can see that various types of dynamics are possible according to the shape of the curves and the domain of the eigenvalues.

Several relevant points on the above dispersion curves are:

a) The extremities of the zone where the eigenvalues are complex conjugated are:

$$\begin{aligned} K_{1D_{left}} &= \frac{\gamma}{D_v - D_u}\left[(f_u - g_v) - 2\sqrt{-f_v g_u}\right] \\ K_{1D_{right}} &= \frac{\gamma}{D_v - D_u}\left[(f_u - g_v) + 2\sqrt{-f_v g_u}\right] \end{aligned} \tag{11}$$

b) The width of the middle zone:

$$\Delta K_{1D} = -\frac{4\gamma\sqrt{-f_v g_u}}{D_u - D_v} \tag{12}$$

c) The coordinates of the center of the middle zone of the dispersion curve are:

$$K_{1Dmiddle} = -\frac{\gamma}{D_u - D_v}(f_u - g_v) \tag{13}$$

and

$$\mathrm{Re}(K_{1Dmiddle}) = \frac{\gamma}{2(D_u - D_v)}(g_v D_u - f_u D_v) \tag{14}$$

From equations (11) and (12) one can easily see that the necessary condition for the middle zone to exist (K_{1Dleft} and $K_{1Dright}$ to be real numbers) is that the product $f_v g_u$ have negative sign. To the limit if $f_v g_u=0$, there is only one value K_{1D} for which the temporal eigenvalues are complex conjugated.

d) When D_u is different of –Dv there are two extreme points, a maximum and a minimum, one on the two branches of the roots of the characteristic polynomial.

$$\begin{aligned} K_{1D_1} &= \frac{\gamma}{D_u - D_v}\left\{g_v - f_u - (D_u + D_v)\sqrt{\frac{-f_v g_u}{D_u D_v}}\right\} \\ K_{1D_2} &= \frac{\gamma}{D_u - D_v}\left\{g_v - f_u + (D_u + D_v)\sqrt{\frac{-f_v g_u}{D_u D_v}}\right\} \end{aligned} \tag{15}$$

From the equations above it is obvious that another necessary condition to have real values is that $f_v g_u D_u D_v < 0$.

These points are useful to obtain band-pass characteristics and band-stop characteristics respectively.

The cases corresponding to D_u=-D_v and respectively to the one when D_uD_v>0.

For both pictures f_vg_u<0.

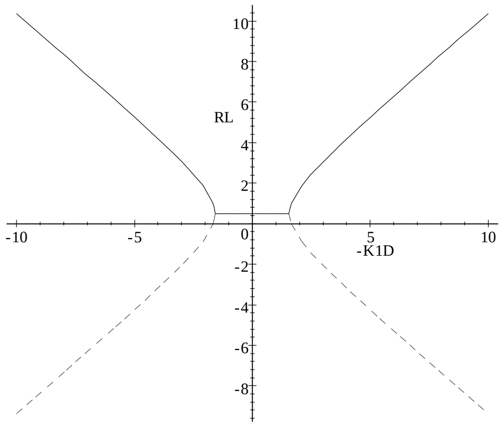

Figure 2: An example for the case D_u=-D_v, f_vg_u<0

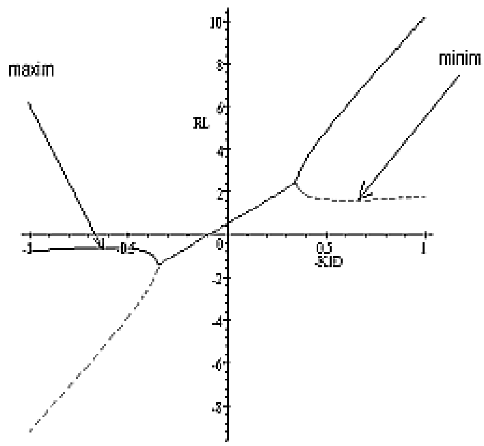

Figure 3: An example for the case D_uD_v>0, f_vg_u<0

e) The real part of the temporal eigenvalue for the zero spatial eigenvalue is:

$$\mathrm{Re}(\lambda_{1,2}(0)) = \gamma f_u \tag{16}$$

Using the above points, it is possible to design a CNN with a specified behavior for a given domain of eigenvalues. The steps to be taken are:

- Depending of the desired dispersion curve one choose the parameters of the system using equations (11-16);
- One chooses a window, which determine the parameters A and B (center and width). See (4)

Now it should be clear that the synthesis of CNN with a specified behavior reduces to the adoption of a window, which is determined by two parameters A (center), and B (width).

4. First order cell – second order neighborhood CNN's

For the central linear part, the set of differential equations is

$$\frac{du_i(t)}{dt} = -u_i + O_{1D}^{r=2}(u_i) + \varepsilon, \quad i = 0..M \tag{17}$$

where, for a symmetric template of the form

C	B	A	B	C

the spatial operator $O_{1D}^{r=2}$ is:

$$O_{1D}^{r=2}(u_i) = Au_i + Bu_{i-1} + Bu_{i+1} + Cu_{i-2} + Cu_{i+2} \tag{18}$$

and the spatial eigenvalues are:

$$K_{1D}^{r=2} = A^{r=2} - 2C^{r=2} + 2B^{r=2}\cos(\omega(m)) + 4C^{r=2}\cos^2(\omega(m)) \tag{19}$$

Recalling the result that for the second order cell – first order neighborhood CNN the eigenvalues are given by (4), in order to make a comparison between the various cases we cast (19) in the form:

$$K_{1D}^{r=2} = \alpha(K_{1D})^2 + \beta K_{1D} + \gamma \tag{20}$$

where K_{1D} is given by (4) where, once A and B have been adopted, the parameters α, β and γ can be determined uniquely in terms of $A^{r=2}$, $B^{r=2}$ and $C^{r=2}$.

For synthesis purposes the template can be determined using the following relations:

$$\begin{cases} A^{r=2} = \alpha(A^2 + 2B^2) + \beta A + \gamma - 1 \\ B^{r=2} = 2\alpha AB + \beta B \\ C^{r=2} = \alpha B^2 \end{cases} \tag{21}$$

With the above notations, the dispersion curve can be written in the form

$$\mathrm{Re}(\lambda(K_{1D})) = \alpha(K_{1D})^2 + \beta K_{1D} + \gamma - 1 \tag{22}$$

It has the shape sketched in Figure 4.

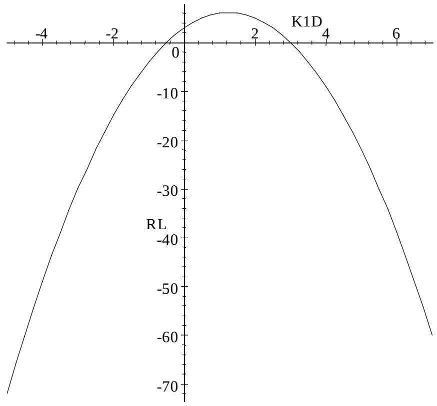

Figure 4: Dispersion curve for α=-2, β=5 γ=4

For α<0, the dispersion curve has a maximum and for α>0, it has a minimum. The extreme values of

$$\mathrm{Re}(\lambda(K_{1D}))\big|\max/\min = -\frac{\beta^2}{4\alpha} + \gamma - 1 \qquad (23)$$

occur for :

$$K_{1Dextrem} = \frac{-\beta}{2\alpha} \qquad (24)$$

Let us observe that γ determines the position of the dispersion curve.

The dispersion curve crosses the x-axis in the points:

$$\begin{aligned} K_{1Dleft} &= \min\left\{\frac{1}{2\alpha}\left(-\beta \pm \sqrt{\beta^2 - 4\alpha(\gamma-1)}\right)\right\} \\ K_{1Dright} &= \max\left\{\frac{1}{2\alpha}\left(-\beta \pm \sqrt{\beta^2 - 4\alpha(\gamma-1)}\right)\right\} \end{aligned} \qquad (25)$$

In order to have at least one mode with positive real part the following condition should be fulfilled:

$$\beta^2 - 4\alpha(\gamma-1) > 0 \qquad (26)$$

5. Concluding remarks

A comparative study of the dynamics of first order cell - second order neighborhood and second order cell – first order neighborhood CNN's is presented.

The shape of the dispersion curve in terms of K_{1D} determines the behavior of the two CNN types. They can be coherently compared on the basis of two template parameters, A and B which determine the center and the width of a window that selects the "active" region of the dispersion curve. From the synthesis point of view, there are to degrees of freedom, one concerning the shape of the dispersion curve (mainly determined by cell parameters) and the position of a window (determined by the template parameters)

Both systems allow low-pass, high-pass and band-pass behavior.

From this point of view (for spatial filtering applications) CNN's made with first order cells and second order neighborhood are easier to use, while second order cell – first order template exhibit more complex dynamics. Indeed, such systems can oscillate in time, a behavior which will be analyzed in a forthcoming paper.

References

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