On the Dynamics of a Class of Cellular Neural Networks

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The stability and dynamics of a class of Cellular Neural Networks (CNN's) in the central linear part is investigated using the decoupling technique based on discrete spatial transforms, Nyquist and root locus techniques.

Introduction

Since their invention [1,2], CNN's - homogeneous arrays of *identical and identically coupled c*ells - have been intensely investigated for their applications in fast image processing [3-9].

The standard CNN cell [1] consists of an input source, an RC parallel circuit, a biasing source and a nonlinear (piecewise linear saturation type) controlled source, which converts the state of the cell into the output. The state of each cell is determined by the outputs (A template) and inputs (B template) of the neighboring cells through controlled sources. The image to be processed with CNN's is introduced through the input and/or the state of the cells, each cell of an N by M array corresponding to a pixel of the image.

An interesting phenomenon [10-17], which has been shown to appear in CNN's, is that of pattern[1] formation - a property that has not been yet enough exploited.

In this communication, the dynamics in the central linear part of a class of 1D CNN's is investigated. The CNN cells are supposed to be nonlinear (piecewise linear) dynamic one-ports, which for the central linear part behave linearly as a one-port admittance Y(s) as shown in Fig. 1.

[1]Pattern will be the name for any stable equilibrium point.

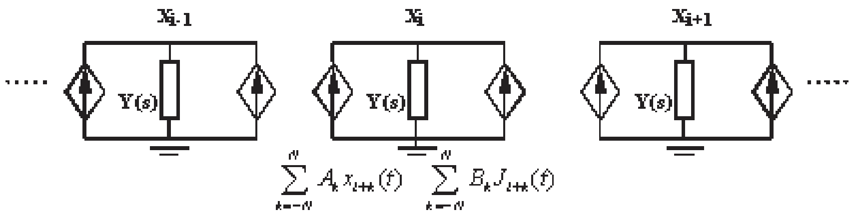


Fig. 1. Sketch of the 1D CNN model for the central linear part.

The CNN is described, in the linear central part, by the set of differential equations .

$$Y(s)x_i(t) = \sum_{k=-N}^{N} A_k x_{i+k}(t) + \sum_{k=-N}^{N} B_k J_{i+k}(t) \tag{1}$$

where s=d/dt, N is the neighborhood radius and A_k and B_k characterize the connection with neighboring cell outputs and the input sources $J_i(t)$, respectively.

In general, $\mathrm{Y(s)} = \frac{\mathrm{Q(s)}}{\mathrm{P(s)}}$ where P(s) and Q(s) are polynomials in the variable s.

Using the decoupling technique, i.e., the change of variables

$$x_i(t) = \sum_{m=0}^{M-1} \Phi_M(i,m)\hat{x}_m(t) \tag{2}$$

and

$$J_i(t) = \sum_{m=0}^{M-1} \Phi_M(i,m)\hat{J}_m(t) \tag{3}$$

i=0,1,…,M-1, with eigenfunctions $\Phi_M(i,m)$ of the form (ring BC's)

$$\Phi_M(i,m) = e^{j\frac{2\pi}{M}mi} \tag{4}$$

in the general case, the action of the A-template gives

$$\sum_{k=-N}^{N} A_k e^{j\frac{2\pi}{M}m(i+k)} = K_A(m) e^{j\frac{2\pi}{M}mi} \tag{5}$$

where,

$$K_A(m) = \sum_{k=-N}^{N} A_k e^{j\frac{2\pi}{M}mk} = \tag{6}$$

$$A_0 + \sum_{k=1}^{N}(A_k + A_{-k})\cos\frac{2mk\pi}{M} + j\sum_{k=1}^{N}(A_k - A_{-k})\sin\frac{2mk\pi}{M}$$

Similar relations are valid for the B-template.

The position of the eigenvalues $K_A(m)$ for several templates corresponding to first and second order neighborhoods are given in Fig. 2.

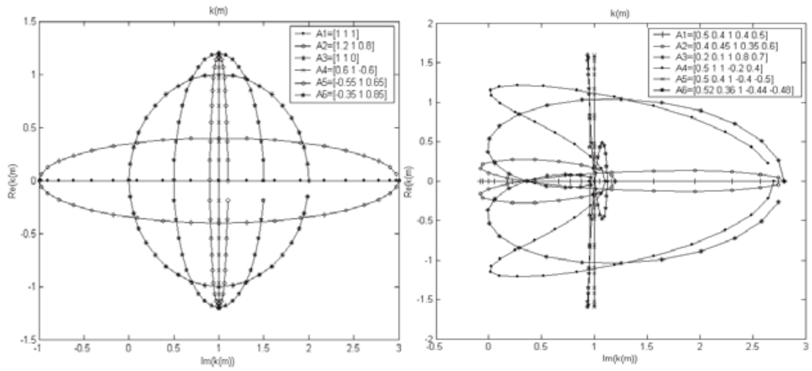


Fig. 2. Eigenvalues position in the complex plane for several first and second order templates.

With the above change of variables, the equations become

$$Y(s)\left(\sum_{n=0}^{M-1}\Phi_M(i,n)\hat{x}_n(t)\right)=\sum_{k=-N}^{N}A_k\left(\sum_{n=0}^{M-1}\Phi_M(i,n)\hat{x}_n(t)\right)+\sum_{k=-N}^{N}B_k\left(\sum_{n=0}^{M-1}\Phi_M(i,n)\hat{J}_n(t)\right) \tag{7}$$

Taking successively the scalar product with $\Phi_M(i,m)$ of both sides the above equations decouple. Thus, each spatial mode, m, is characterized by the differential equation

$$Y(s)\hat{x}_m(t)=K_A(m)\hat{x}_m(t)+K_B(m)\hat{J}_m(t) \tag{8}$$

and the transfer function

$$H_m(s)=\frac{K_B(m)}{\dfrac{Q(s)}{P(s)}-K_A(m)} \tag{9}$$

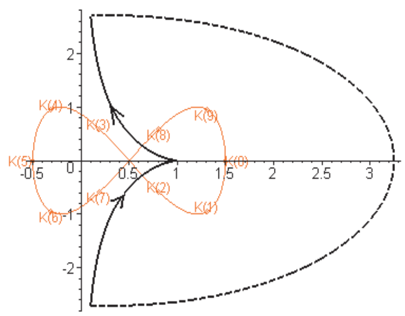
The decoupling technique allows the study of the CNN dynamics based on the time evolution of each spatial mode. Patterns can be produced by the input when the CNN is stable and by both the input and the state when at least one spatial mode is unstable. In such cases the node voltages will grow until limited by some nonlinear mechanism.

The stability and dynamics of each spatial mode are determined by the zeros of $\dfrac{Q(s)}{P(s)}-K_A(m)$. Taking advantage of the special form of this equation, the stability and dynamics of the CNN can be studied using techniques from feedback theory, i.e., the Nyquist criterion and the root locus method adapted for complex amplifications as, in the general case of a asymmetric templates, ($A_i \neq A_{-I}$) K_A(m) is complex and satisfies the condition $K_A(m)=K_A^*(M-m)$.

The Nyquist technique

The use of the Nyquist criterion for mode stability investigation is based on the observation that the stability depends on the relative position of $K_A(m)$ with respect to the hodograph of Q(s)/P(s) for s belonging to the Nyquist contour. The stability of the modes is determined by the number of times $K_A(m)$ and $K_A^*(m)$ are circled by the hodograph of Y(s). Thus, the shape of the hodograph of Y(s) gives the influence of the cell on the dynamics while the template influence is given by the position of the eigenvalues with respect to the hodograph. Several examples are given Fig.3 for a CNN made of cells with the admitance $Y(s)=(s^2+s+1)/(s+1)$, J(t)=0. As expected, looking at the hodograph and the real parts of the temporal eigenvalues, the patterns developed from random initial conditions were either mode *zero* or mode *one*, oscillating in time (Fig. 3. a-d).

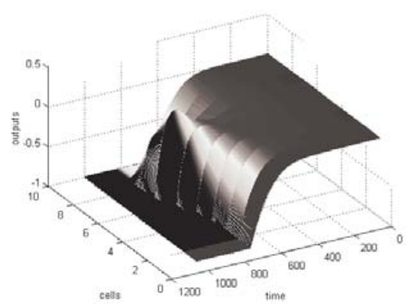
Similarly, in Fig. 3. e,f and Fig. 3. g,h the final pattern are again those predicted from considerations related to the real part of the temporal eigenvalues calculated for the central linear part (modes *three* and *four* respectively).



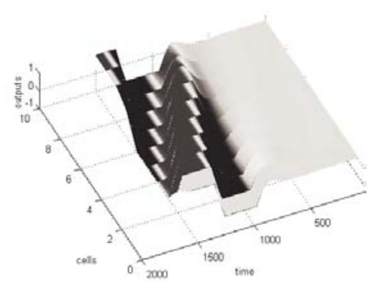
a.

Mode	Temporal eigenvalues	
0	1	-0.5
1	0.93+1.18i	-0.62-0.23i
2	0.31+0.95i	-0.5-0.36i
3	-0.21-1.17i	-0.6+0.58i
4	-0.5-1.5i	-0.81+0.59i
5	-0.75+0.97i	-0.75-0.97i

b.



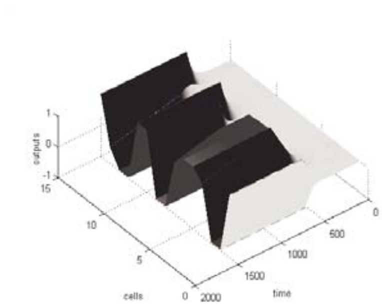
c.



d.

Mode	Temporal eigenvalues	
0	0	0
1	0.63	-0.39
2	1.35	-0.57
3	1.75	-0.64
4	1.47	-0.59
5	0	0
6	-0.56+0.9i	-0.56-0.9i
7	-0.95+i	-0.95-i

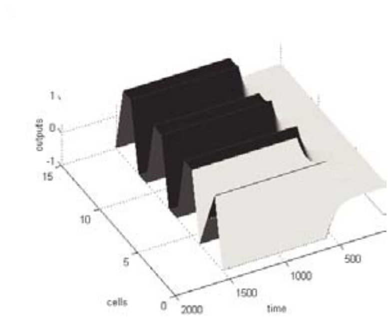
e.



f.

Mode	Temporal eigenvalues	
0	-1+i	-1-i
1	-0.79+0.98i	-0.79-0.98i
2	-0.23+0.7i	-0.23-0.7i
3	1	-5
4	1.7	-0.63
5	1.62	-0.62
6	1	-5
7	0.29	-0.22

g.



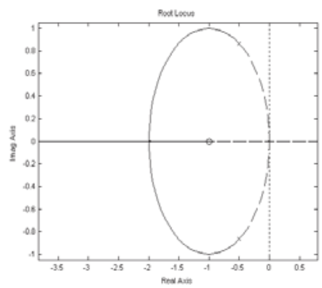
h.

Fig. 3 Hodograph of Y(s) and spatial eigenvalues (**a**); Spatial modes and their temporal eigenvalues for 10 cells, A=[-0.5 0.5 0.5 0.5 0.5] (**b**); Output evolution for random initial conditions with different seeds (**c,d**); Modes, temporal eigenvalues and output evolution for random initial conditions for 15 cells, A=[-0.5 0.5 1 0.5 -0.5] (**e**,**f**) and A=[-0.5 –0.5 1 –0.5 -0.5] (**g, h**).

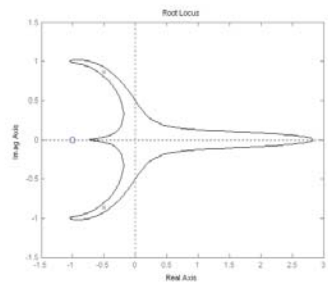
The root locus technique

The root locus technique can be used as well, especially for symmetric templates when the eigenvalues are real: The roots of the characteristic equations are placed on the branches of the root locus in a window whose center is determined by A_0 and the width by the other template coefficients. The branches of the root locus within a given “window” are not followed in one direction as it happens in the classical cases when the amplification varies monotonically from zero to infinity. Indeed, the portion of the root locus corresponding to the eigenvalues can be followed in both directions according to the template. Using this tool combined with the window method it is easy to make an image of the array dynamics in the central linear part.

For asymmetric templates the shape of the root locus changes depending on the asymmetry of the template. Several root loci are presented in Figs.4 and 5.



$Y(s)=(s^2+s+1)/(s+1)$,
A=[1 1 1]



Y(s)=(s2+s+1)/(s+1),
A=[1 1 1.1]

Fig.4 Root loci for a symmetric and an asymmetric template

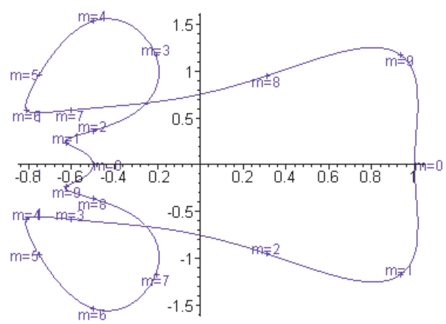


Fig. 5. Root locus coresponding to the case of Fig. 3 a-b.

Concluding remarks

In this work, several analytical results concerning the dynamics of CNN and pattern formation have been presented. They are based on the decoupling technique and represent a generalization of previously reported techniques used for Turing pattern formation. The method has been combined with Nyquist and root locus techniques and puts into evidence the dynamics of the CNN with respect to the Fourier components of the input and initial conditions for ring boundary. Although intrinsically linear, the approach gives significant insight for pattern formation.

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