Iteration Bound

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Outline

- *Introduction*
- Data Flow Graph (DFG) Representations
- Loop Bound and Iteration Bound
- Compute the Iteration Bound
  - Longest Path Matrix Algorithm (LPM)
  - Minimum Cycle Mean Method (MCM)
- Conclusion

Introduction

- Iteration bound = maximum loop bound OR maximum non-loop bound
- Clock period = critical path period = cycle time = 1/clock rate
- Sample rate = throughput rate
- Impossible to achieve an iteration bound less than the theoretical iteration bound with infinite processors

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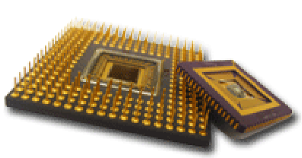
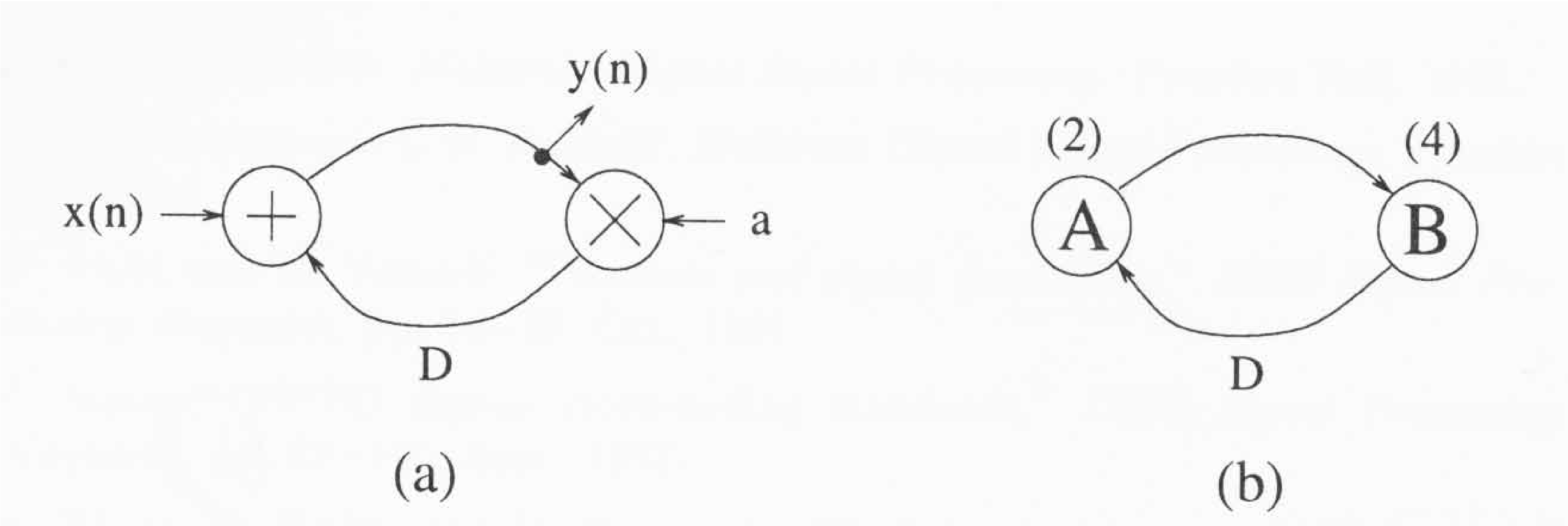


DSP Program: Example

$$\text{for } n = 0 \ \ to \ \ \infty$$

$$y(n) = ay(n-1) + x(n)$$



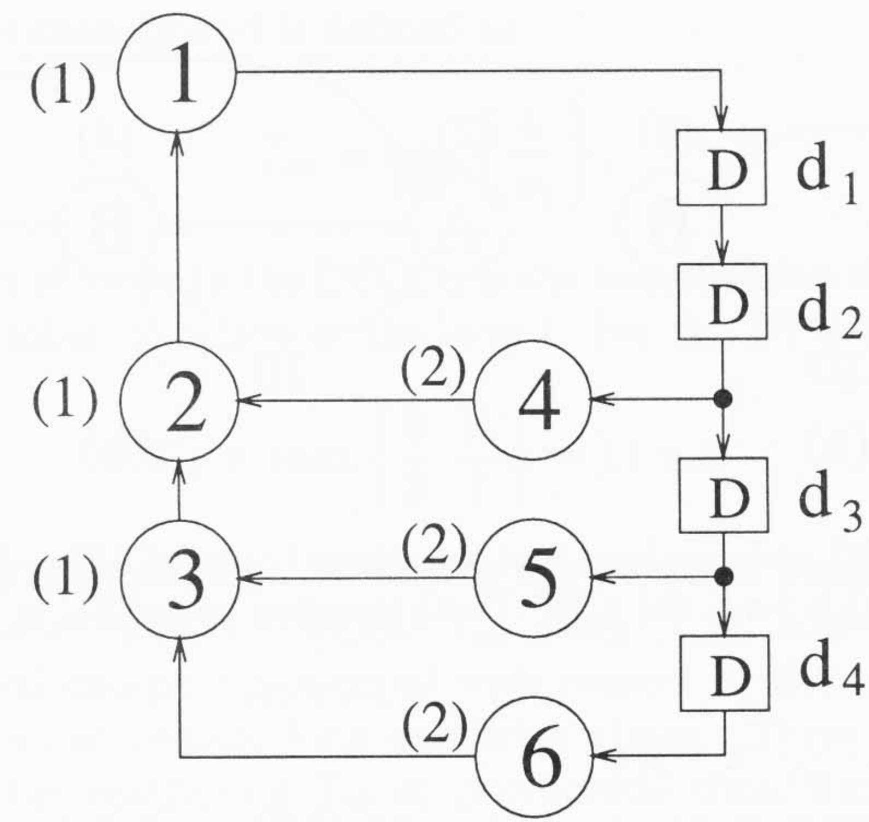
Data Flow Graph

$$\text{Intra - Iteration Period}: A_k \rightarrow B_k$$

$$\text{Inter - Iteration Period}: B_k \Rightarrow A_{k+1}$$

Loop Bounds in DFG

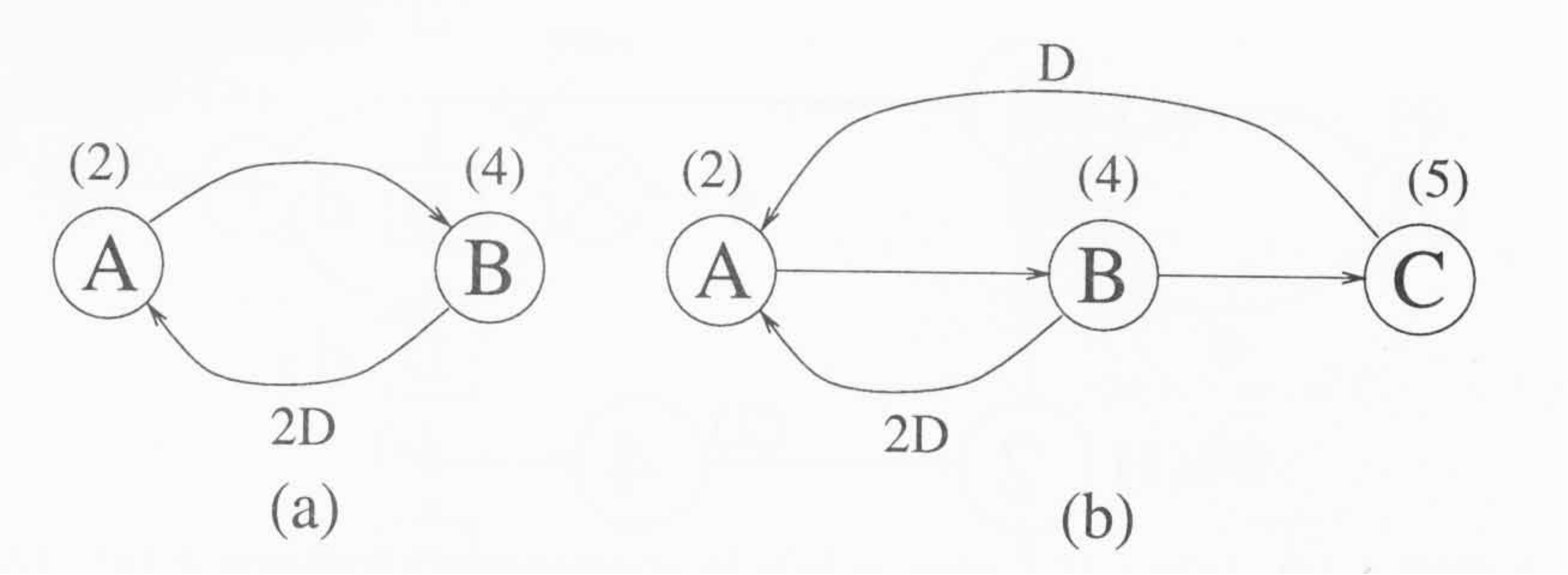
- Critical Path: The path with the longest computation time among all paths that contain zero delays
- Loop: Directed path that begins and ends at the same node
- Loop Bound=t_l/w_l, where t_l is the loop computation time and w_l is the number of delays in the loop
- Critical Loop: the loops in which has maximum loop bound.
- Iteration Bound: maximum loop bound / non-loop bound, i.e., a fundamental limit for recursive / non-recursive algorithms



Loop bounds : 4/2 u.t. (max), 5/3 u.t. , 5/4 u.t.

Iteration bound

Definition: $$T_\infty = \max_{l \in L}\left\{\frac{t_l}{w_l}\right\}$$



(a) Iteration bound= 6/2 = 3 u.t.

(b) Iteration bound= Max { 6/2 , 11/1 } = 11 u.t.

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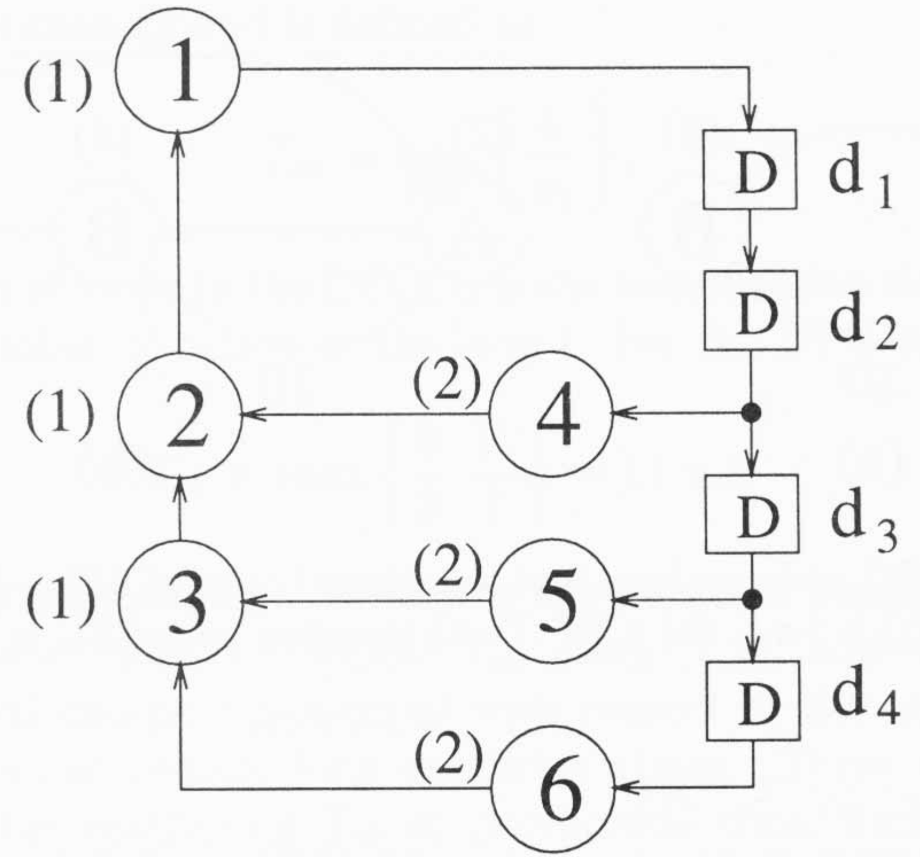
Longest Path Matrix Algorithm (LPM)

- A series of matrix is constructed, and the iteration bound is found by examining the diagonal elements of the matrices.
- d : # of delays
- Compute L1~Lm, m=1, 2, 3, …, d
- $l_{i,j}^{(m)}$:The longest computation time of all paths from delay element d_i to delay element d_j that pass through exactly m-1 delays, where the delay d_i and delay d_j are not included for m-1 delays.
- If no path exists, then the value of *l* equals -1.

A DFG with Three Loops Using LPM (1/3)

$$\mathbf{L}^{(1)} = \begin{bmatrix} -1 & 0 & -1 & -1 \\ 4 & -1 & 0 & -1 \\ 5 & -1 & -1 & 0 \\ 5 & -1 & -1 & -1 \end{bmatrix}$$

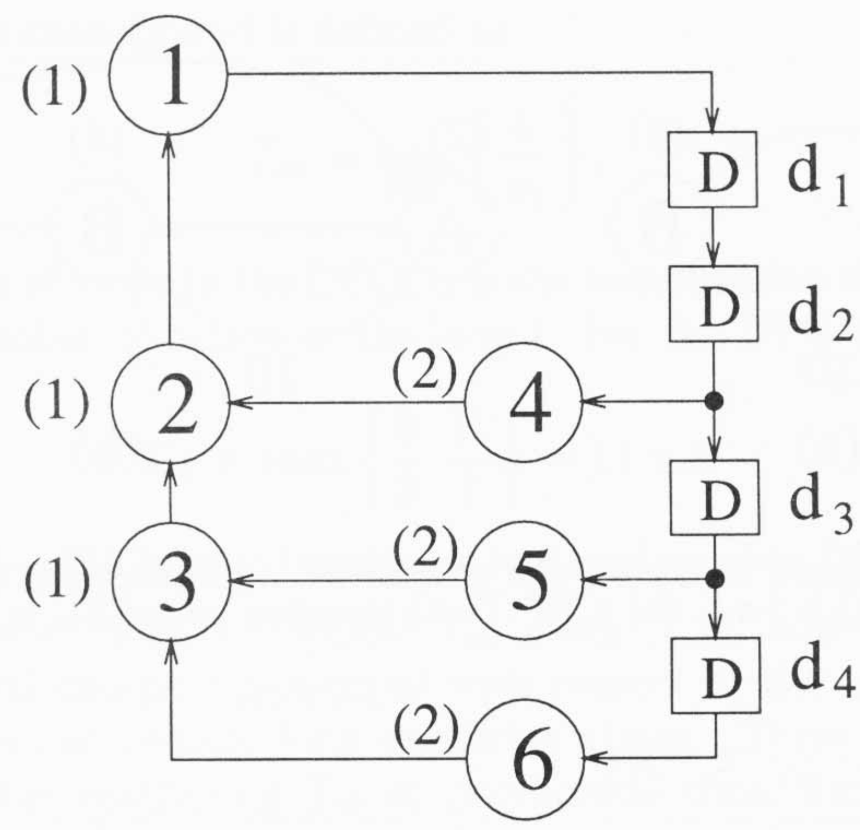


A DFG with Three Loops Using LPM (2/3)

$$l_{i,j}^{(m+1)} = \max_{k \in K}\left(-1, l_{i,k}^{(1)} + l_{k,j}^{(m)}\right)$$

$$\mathbf{L}^{(2)} = \begin{bmatrix} -1 & 0 & -1 & -1 \\ 4 & -1 & 0 & -1 \\ 5 & -1 & -1 & 0 \\ 5 & -1 & -1 & -1 \end{bmatrix} \times \begin{bmatrix} -1 & 0 & -1 & -1 \\ 4 & -1 & 0 & -1 \\ 5 & -1 & -1 & 0 \\ 5 & -1 & -1 & -1 \end{bmatrix}$$

$$\mathbf{L}^{(2)} = \begin{bmatrix} 4 & -1 & 0 & -1 \\ 5 & 4 & -1 & 0 \\ 5 & 5 & -1 & -1 \\ -1 & 5 & -1 & -1 \end{bmatrix}$$

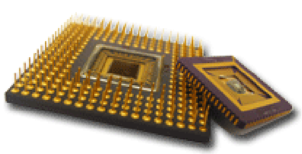


A DFG with Three Loops Using LPM (3/3)

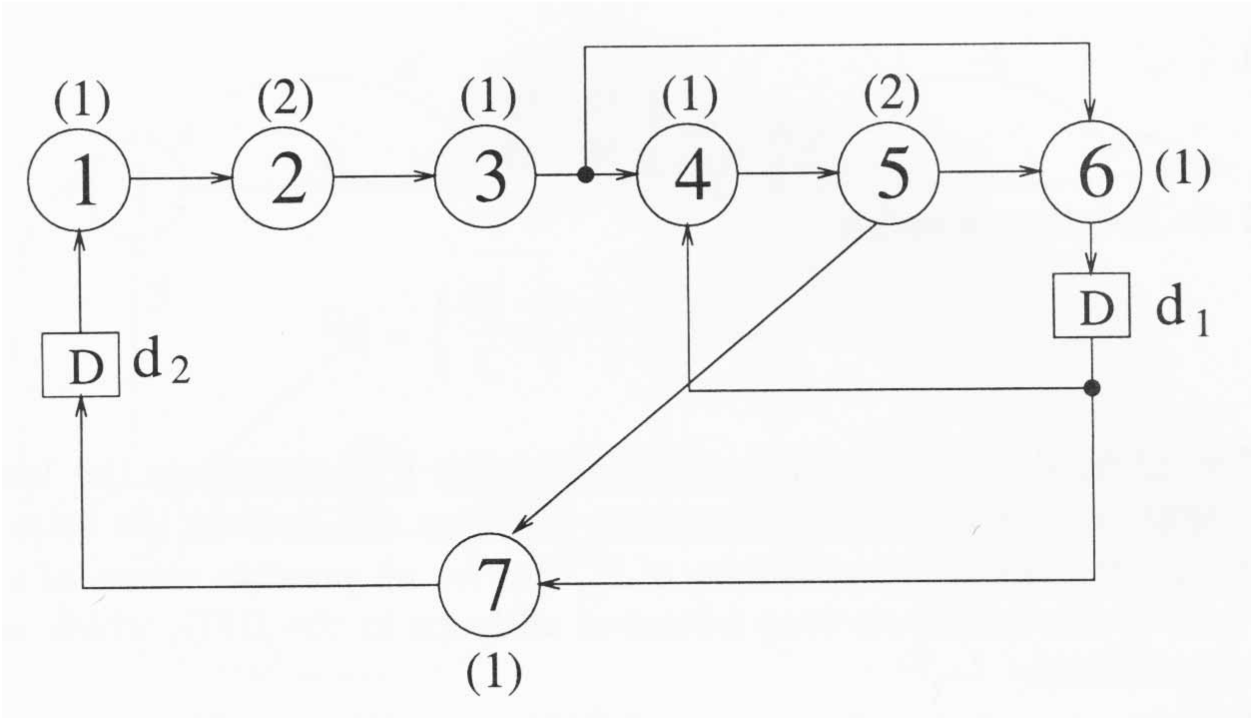
$$\mathbf{L}^{(3)} = \begin{bmatrix} 5 & 4 & -1 & 0 \\ 8 & 5 & 4 & -1 \\ 9 & 5 & 5 & -1 \\ 9 & -1 & 5 & -1 \end{bmatrix} \qquad \mathbf{L}^{(4)} = \begin{bmatrix} 8 & 5 & 4 & -1 \\ 9 & 8 & 5 & 4 \\ 10 & 9 & 5 & 5 \\ 10 & 9 & -1 & 5 \end{bmatrix}$$

$$T_\infty = \max_{i,m\in\{1,2,\ldots,d\}} \left\{ \frac{l_{i,i}^{(m)}}{m} \right\}$$

$$T_\infty = \max_{i,m\in\{1,2,\ldots,d\}} \left\{ \frac{4}{2}, \frac{4}{2}, \frac{5}{3}, \frac{5}{3}, \frac{5}{3}, \frac{8}{4}, \frac{8}{4}, \frac{5}{4}, \frac{5}{4} \right\} = 2$$



A Filter Using LPM



$$\mathbf{L}^{(1)} = \begin{bmatrix} 4 & 4 \\ 8 & 8 \end{bmatrix} \qquad \mathbf{L}^{(2)} = \begin{bmatrix} 12 & 12 \\ 16 & 16 \end{bmatrix}$$

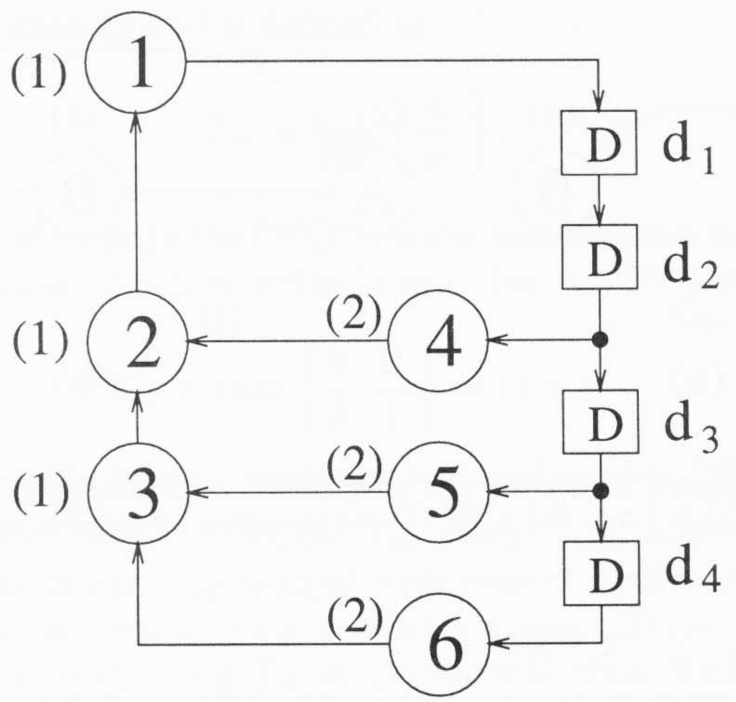
$$T_\infty = \max\{\frac{4}{1}, \frac{8}{1}, \frac{12}{2}, \frac{16}{2}\} = 8$$

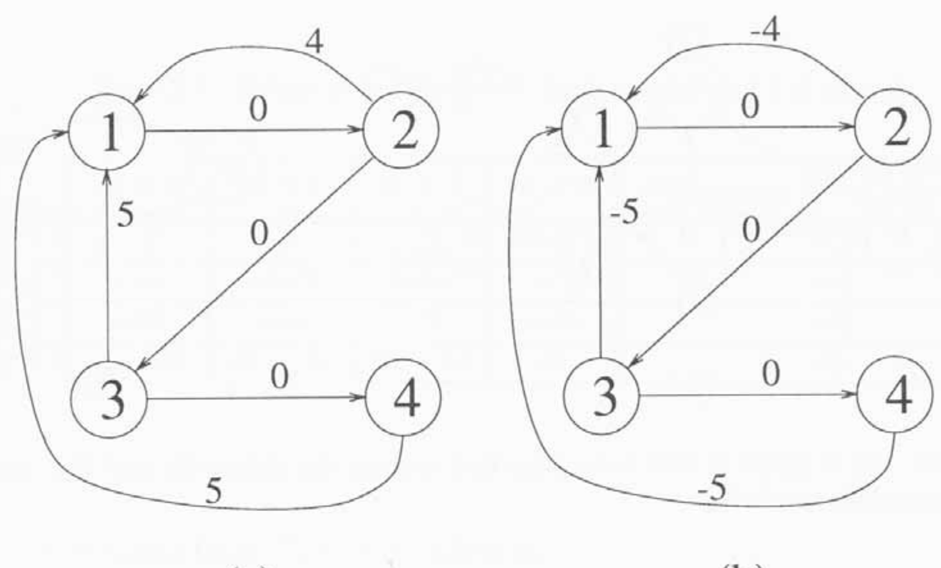
Minimum Cycle Mean Method (MCM)

- 1. Construct the new graph G_d (DFG G)
  - transform from DFG
  - decide the weight of each edge
- 2. Compute the maximum cycle mean
  - construct the series of d+1 vectors $f^{(m)}$, *m=0, 1, 2, …,d*
    - *An arbitrary reference node is chosen in* *<u>Gd</u>* *(called this node s). The initial vector* **f**$^{(0)}$ is formed by setting $f^{(0)}(s)=0$ and setting the remaining nodes of **f**$^{(0)}$ to infinity.
  - find the max cycle mean
- 3. Find the min cycle mean between each cycle

A DFG with Three Loops Using MCM (1/5)

- delay => node
- longest path length (computation time) =>weight w(i,j)
  - If no zero-delay path exists from delay d_i to delay d_j, then the edge I -> j does not exist in Gd.

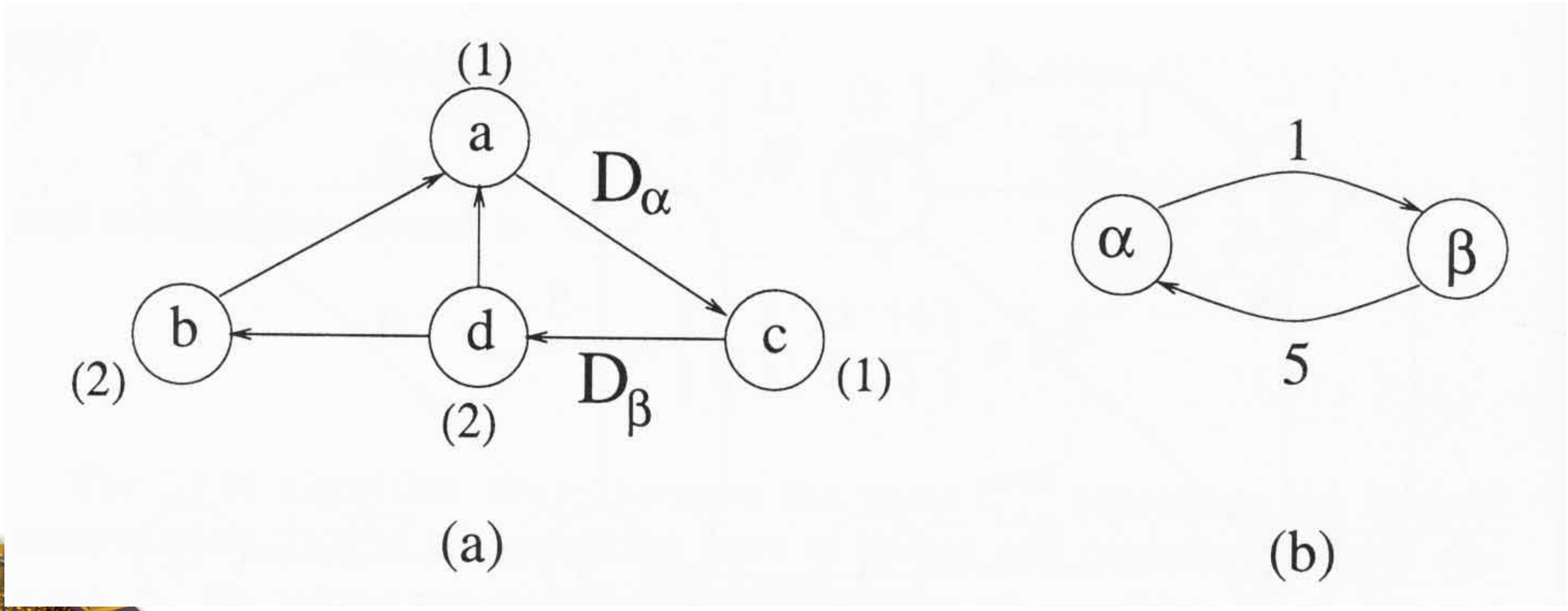




(a) (b)

A DFG with Three Loops Using MCM (2/5)

- Longest path length
  - path which pass through no delays
  - longest : two loops that contain D_a and Db
    - max { 6,4 } = 6
  - cycle mean = 6/2=3



A DFG with Three Loops Using MCM (3/5)

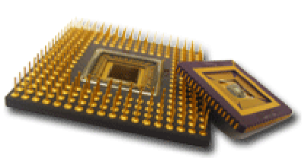
- Cycle mean= Average length of the edge in c (Cycle = Loop)
- Compute the cycle mean

A DFG with Three Loops Using MCM (4/5)

- we will find d+1 vectors , $f^{(m)}$ m=0,1,…, d

$$f^{(0)} = \begin{bmatrix} 0 \\ \infty \\ \infty \\ \infty \end{bmatrix} \quad f^{(1)} = \begin{bmatrix} \infty \\ 0 \\ \infty \\ \infty \end{bmatrix} \quad f^{(2)} = \begin{bmatrix} -4 \\ \infty \\ 0 \\ \infty \end{bmatrix} \quad f^{(3)} = \begin{bmatrix} -5 \\ -4 \\ \infty \\ 0 \end{bmatrix} \quad f^{(4)} = \begin{bmatrix} -8 \\ -5 \\ -4 \\ \infty \end{bmatrix}$$

$$f^{(m)}(j) = \min_{i \in I}\left(f^{(m-1)}(i) + \overline{w}(i,j)\right)$$



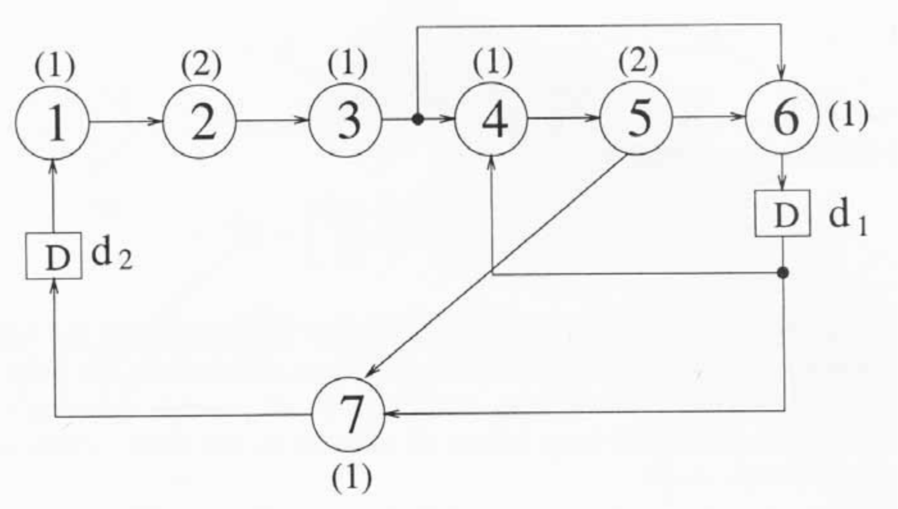
A DFG with Three Loops Using MCM (5/5)

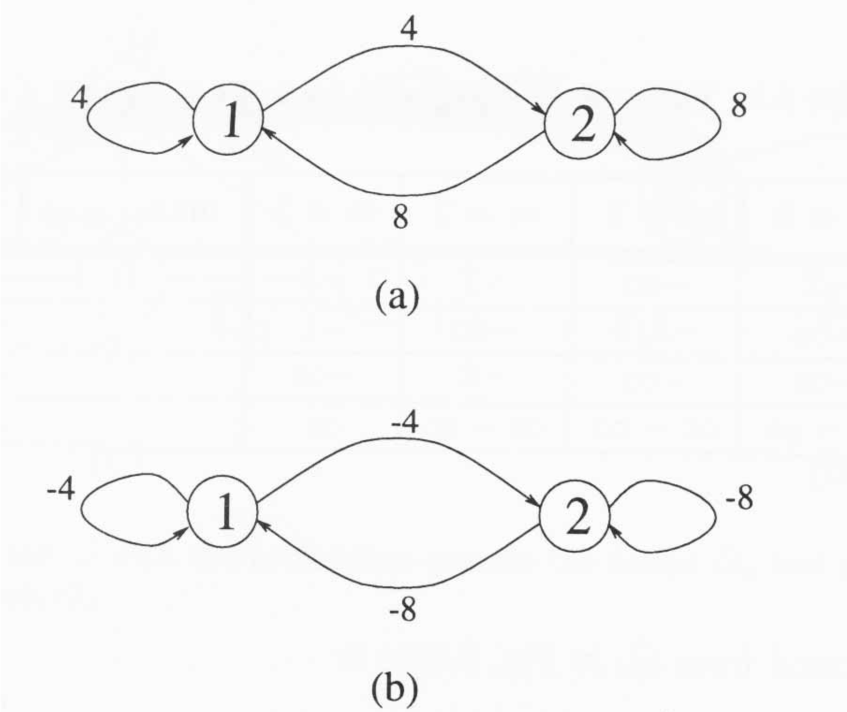
$$T_\infty = -\min_{i\in\{1,2,\ldots,d\}}\left(\max_{m\in\{0,1,2,\ldots d-1\}}\left(\frac{f^{(d)}(i)-f^{(m)}(i)}{d-m}\right)\right)$$

	$m=0$	$m=1$	$m=2$	$m=3$	$\max_{0\le m\le 3}\left\{\frac{f^{(4)}(i)-f^{(m)}(i)}{4-m}\right\}$
$i=1$	-2	$-\infty$	-2	-3	-2
$i=2$	$-\infty$	$-5/3$	$-\infty$	-1	-1
$i=3$	$-\infty$	$-\infty$	-2	$-\infty$	-2
$i=4$	$\infty-\infty$	$\infty-\infty$	$\infty-\infty$	∞	∞

$$T_\infty = -\min\{-2,-1,-2,\infty\} = 2$$

A Filter Using MCM





$$f^{(0)} = \begin{bmatrix} 0 \\ \infty \end{bmatrix} \quad f^{(1)} = \begin{bmatrix} -4 \\ -4 \end{bmatrix} \quad f^{(2)} = \begin{bmatrix} -12 \\ -12 \end{bmatrix}$$

	$m=0$	$m=1$	$\max_{0\le m\le 1}\left\{\frac{f^{(2)}(i)-f^{(m)}(i)}{2-m}\right\}$
$i=1$	$-12/2$	$-8/1$	-6
$i=2$	$-\infty$	$-8/1$	-8

$$T_\infty = -\min\{-8,-6\} = 8$$

Conclusion

- When the DFG is recursive, the iteration bound is the fundamental limit on the minimum sample period of a hardware implementation of the DSP program.
- Two algorithms to compute iteration bound, LPM and MCM, were explored.

Self-Test Exercises

- STE1: For the DFG shown in Fig. 2.12, the computation times of the nodes are shown in parentheses. Compute the iteration bound of this DFG using (a) the LPM algorithm, and (b) the MCM algorithm. (Also see Problem 2.1 of the text book.)
- STE2: Repeat STE1 for the DFG shown in Fig. 2.15 assuming that addition and multiplication require 1 and 2 u.t., respectively. (Also see Problem 2.4 of the text book.)